# PREDICTION OF FIRM-LEVEL TECHNICAL EFFICIENCIES WITH A GENERALIZED FRONTIER PRODUCTION FUNCTION AND PANEL DATA\*

## George E. BATTESE and Tim J. COELLI

University of New England, Armidale, NSW, 2351, Australia

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A stochastic frontier production function is defined for panel data on sample firms, such that the disturbances associated with observations for a given firm involve the differences between traditional symmetric random errors and a non-negative random variable, which is associated with the technical efficiency of the firm. Given that the non-negative firm effects are time-invariant and have a general truncated normal distribution, we obtain the best predictor for the firm-effect random variable and the appropriate technical efficiency of an individual firm, given the values of the disturbances in the model. The results obtained are a generalization of those presented by Jondrow et al. (1982) for a cross-sectional model in which the firm effects have half-normal distribution. The model is applied in the analysis of three years of data for dairy farms in Australia.

### 1. Introduction

The stochastic frontier production function, proposed independently by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977), has been considered and applied or modified in a number of studies, including Battese and Corra (1977), Lee and Tyler (1978), Stevenson (1980), Pitt and Lee (1981), Jondrow et al. (1982), Kalirajan (1982), Bagi and Huang (1983), Kalirajan and Flinn (1983), Huang and Bagi (1984), Schmidt and Sickles (1984), Waldman (1984) and Coelli (1985). The earlier studies involved the estimation of the parameters of the stochastic frontier production function and the mean technical efficiency for firms in the industry. It was initially claimed that technical efficiencies for individual sample firms could not be predicted. Jondrow et al. (1982) presented two predictors for the firm effect for an individual firm on the assumption that the parameters of the frontier production function were known and cross-sectional data were available for sample firms. Schmidt and Sickles (1984) considered a number of methods of predic-

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ting individual firm effects (and hence, technical efficiencies) given that panel data were available on sample firms. Waldman (1984) investigated the properties of a predictor for firm technical efficiencies proposed by Jondrow et al. (1982) and two other possible predictors.

In this paper we present a generalization of some of the results presented by Jondrow et al. (1982), under the assumption that panel data on sample firms are available and that a more general distribution for firm effects, suggested by Stevenson (1980), applies for the stochastic frontier production function.

# 2. The frontier production function

Consider the frontier production function

$$Y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + E_{it},\tag{1}$$

and

$$E_{it} = V_{it} - U_i, \tag{2}$$

where  $Y_{ii}$  denotes the appropriate function (e.g., logarithm) of the production for the *i*th sample firm (i = 1, 2, ..., N) in the *t*th time period (t = 1, 2, ..., T);  $x_{ii}$  is a  $(1 \times k)$  vector of appropriate functions of the inputs associated with the *i*th sample firm in the *t*th time period (the first element would generally be one);  $\beta$  is a  $(k \times 1)$  vector of the coefficients for the associated independent variables in the production function; the  $V_{ii}$ -random random variables are assumed to be independent and identically distributed as  $N(0, \sigma_V^2)$  independent of the  $U_i$ -random variables, which are assumed to be independent and identically distributed non-negative random variables, defined by the truncation (at zero) of the  $N(\mu, \sigma^2)$  distribution. In addition, it is assumed that the  $V_{ii}$  and  $U_i$ -random variables are independently distributed of the input variables in the model.

The density function for  $U_i$  is defined by

$$f_{U_i}(u) = \frac{\exp\left[-\frac{1}{(2\pi)^{1/2}\sigma}\left[1 - \frac{\Phi(-\mu/\sigma)}{(2\pi)^{1/2}\sigma}\right]}{(2\pi)^{1/2}\sigma\left[1 - \frac{\Phi(-\mu/\sigma)}{(2\pi)^{1/2}\sigma}\right]}, \quad u > 0,$$
(3)

where  $\Phi(\cdot)$  denotes the distribution function of the standard normal random variable.

The distribution of the non-negative firm-effect random variables is that suggested by Stevenson (1980), which is the generalization of the half-normal distribution (in which  $\mu = 0$ ). Pitt and Lee (1981) and Schmidt and Sickles (1984) considered the special case of this model in which the firm effects had half-normal distribution. As noted by Pitt and Lee (1981, p. 46) and Schmidt (1985, p. 314), firms may discover, after a period of time, the extent of their inefficiency and adjust their input values accordingly. This is not assumed to be the case in this paper.

Schmidt (1985, p. 313) states that 'unchanging inefficiency over time is not a particularly attractive assumption, but on the other hand it is a powerful one'. Schmidt (1985, p. 315) also states that 'an important line of future research, in my opinion, is to allow inefficiency to change over time...'. Forsund (1985, p. 333) comments that application and testing of the panel data models for Swedish dairy farms is currently under way. Cornwell, Schmidt and Sickles (1987), in their empirical analysis of twelve years of quarterly data on U.S. airline companies, consider a frontier model in which the firm-effect random disturbances are a quadratic function of time.

Our application of the frontier model (1)-(2) involves only three years of data on Australian dairy farms and so we believe that the time-invariant model for the firm effect random disturbances is not unreasonable.

The likelihood function for observations on the frontier production function (1)-(2) is presented in Coelli (1985) together with its first derivatives, which are required for obtaining approximate maximum-likelihood estimates for the parameters of the model.

#### 3. Firm technical efficiency

We define the technical efficiency of a given firm as the ratio of its mean production (in *original units*), given its realized firm effect, to the corresponding mean production if the firm effect was zero. Thus, the technical efficiency of the *i*th firm, denoted by  $TE_i$ , is defined by

$$TE_{i} = \frac{E(Y_{it}^{*}|U_{i}, x_{it}, t = 1, 2, ...)}{E(Y_{it}^{*}|U_{i} = 0, x_{it}, t = 1, 2, ...)},$$
(4)

where  $Y_{it}^*$  denotes the value of production (in original units) for the *i*th firm in the *t*th time period.

This measure necessarily has values between zero and one. If a firm's technical efficiency is 0.85, then it implies that the firm realizes, on average, 85 percent of the production possible for a fully efficient firm having comparable input values.

If the frontier production function (1)-(2) is defined directly in terms of the original units of production, then the technical efficiency of the *i*th firm is

$$TE_i = (\bar{\mathbf{x}}_i \boldsymbol{\beta} - U_i) (\bar{\mathbf{x}}_i \boldsymbol{\beta})^{-1}, \qquad (5)$$

where  $\bar{x}_i$  represents the mean of the input levels for the *i*th firm. The corresponding measure of (mean) technical efficiency of the firms in the industry, denoted by TE, is given by

$$TE = 1 - \left\{ \mu + \frac{\sigma\phi(-\mu/\sigma)}{1 - \Phi(-\mu/\sigma)} \right\} (\bar{x}\beta)^{-1},$$
(6)

where  $\phi(\cdot)$  represents the density function for the standard normal random variable and  $\bar{x}$  is the mean of the input levels for the firms in the industry.

If the frontier production function (1)-(2) is defined for the logarithm of production, then the production for the *i*th firm in the *t*th period is  $\exp(Y_{it})$ . The suggested measure of technical efficiency for the *i*th firm is thus

$$TE_i = \exp(-U_i). \tag{7}$$

This measure of technical efficiency is equivalent to the ratio of the production for the *i*th firm in any given period t,  $\exp(Y_{it}) = \exp(x_{it}\beta + V_{it} - U_i)$ , to the corresponding production value if the firm effect  $U_i$  was zero,  $\exp(x_{it}\beta - V_{it})$ . The technical efficiency measure (7) is not dependent on the level of the factor inputs for the given firm, which is not the case for the technical efficiency measure (5).

The mean technical efficiency of firms in the industry that corresponds to the measure of (7) is

$$TE = \left\{ \frac{1 - \Phi[\sigma - (\mu/\sigma)]}{1 - \Phi(-\mu/\sigma)} \right\} \exp\left(-\mu + \frac{1}{2}\sigma^2\right).$$
(8)

When  $\mu = 0$ , the mean technical efficiency (8) is equal to that derived by Lee and Tyler (1978, p. 387).

It is important to clearly define the appropriate measure of technical efficiency before reporting numerical values. It appears that there is a degree of confusion in the literature on this point. For example, Jondrow et al. (1982), in their discussion of empirical results reported by Schmidt and Lovell (1980) for the U.S. steam-electric generating plants, state that '... the estimated average technical inefficiency (mean of u) is 0.0959, *indicating about 9.6 percent technical inefficiency*' (p. 236; our italics). Thus Jondrow et at. (1982) estimate technical inefficiency of firms in the industry by estimating the mean of the firm effects [defined by  $u_i$  in Jondrow et al. (1982), but by  $U_i$  above]. However, predicting the technical efficiency of the *i*th firm by predicting the value of the random variable,  $1 - U_i$ , rather than  $\exp(-U_i)$ , as suggested in (7) for the logarithmic case, is not recommended. The expression  $1 - U_i$  includes only the first term in the power-series expansion of  $\exp(-U_i)$ . The remainder term may be significant when the firm effect  $U_i$  is not close to zero.

Given that the frontier production function was stated in logarithmic form in Schmidt and Lovell (1980, p. 84), the mean technical efficiency for the U.S. steam-electric generating plants is

$$TE = 2[1 - \Phi(\sigma)] \exp(\frac{1}{2}\sigma^2),$$

for the half-normal model [cf. eq. (8) above]. The estimate for  $\sigma^2$ , reported by Schmidt and Lovell (1980, p. 90), was 0.014452. [Note that  $\sigma^2$  in our paper

corresponds to  $\sigma_u^2$  in Schmidt and Lovell (1980).] Thus, the appropriate estimate for the mean technical efficiency is

$$\widehat{TE} = 2[1 - \Phi(0.12)]\exp(0.007226) = 0.911.$$

Thus, firms in the industry are about 91.1 percent technically efficient (or 8.9 percent technically inefficient). While this value is close to the estimated value of the mean of the firm effect  $U_i$  for this application, we believe that it is important to evaluate the appropriate measure of technical efficiency.

Given the definition (4) of the technical efficiency of a firm, it is evident that its prediction depends on inference about the appropriate function of the unobservable firm effect  $U_i$ , given the sample observations. We obtain the conditional distribution of the firm effect  $U_i$ , given the values of the random variables,  $E_{it} \equiv V_{it} - U_i$ , t = 1, 2, ..., T. This assumes that the values of the parameter  $\beta$  are known.

Theorem 1. Given the specifications of the frontier production function (1)-(2)and sample values of the random vector,  $E_i \equiv (E_{i1}, E_{i2}, \ldots, E_{iT})'$ , denoted by  $e_i \equiv (e_{i1}, e_{i2}, \ldots, e_{iT})'$ , then the conditional distribution of  $U_i$ , given  $E_i = e_i$ , is defined by the truncation (at zero) of the normal distribution with mean

$$\mu_i^* \equiv \left(-\sigma^2 \bar{\varphi}_i + T^{-1} \mu \sigma_V^2\right) \left(\sigma^2 + T^{-1} \sigma_V^2\right)^{-1}.$$
(9)

and variance

$$\sigma_*^2 \equiv \sigma^2 \sigma_\nu^2 \left( \sigma_\nu^2 + T \sigma^2 \right)^{-1}, \tag{10}$$

where

., ÷

$$\bar{e}_i = T^{-1} \sum_{t=1}^T e_{it}.$$

Further, the conditional expectations of  $U_i$  and  $\exp(-U_i)$ , given  $E_i = e_i$ , are given by

$$\mathbf{E}(U_i \mid E_i = e_i) = \mu_i^* + \sigma_* \left\{ \phi(-\mu_i^* / \sigma_*) \left[ 1 - \Phi(-\mu_i^* / \sigma_*) \right]^{-1} \right\}, \quad (11)$$

and

$$E[\exp(-U_i) | E_i = e_i] = \left\{ \frac{1 - \Phi[\sigma_* - (\mu_i^3 / \sigma_*)]}{1 - \Phi(-\mu_i^* / \sigma_*)} \right\} \exp(-\mu_i^* + \frac{1}{2}\sigma_*^2).$$
(12)

The proof of Theorem 1 involves straightforward, but tedious, algebra.

It is evident that the mean  $\mu_i^*$ , defined by (9), is of order one and converges, as  $T \to \infty$ , to the limit  $\lim_{T\to\infty} -T^{-1}\sum_{i=1}^{T} e_{ii} = u_i$ , which is the sample value of the random effect  $U_i$  for the *i*th firm. Further, the variance  $\sigma_*^2$  defined by (10) is of order  $T^{-1}$  and converges to zero as  $T \to \infty$ .

It is readily seen that the results of Jondrow et al. (1982) for the half-normal case and cross-sectional data are obtained by substituting  $\mu = 0$  and T = 1 in eqs. (9), (10) and (11). Jondrow et al. (1982) did not, however, obtain an expression for the conditional expectation of  $\exp(-U_i)$ , given sample values of  $E_i$ .

Given that the parameters of the frontier production function (1)-(2) are known and the model is defined in terms of the original units of production, then a predictor for the random variable  $U_i$  in the technical efficiency of the *i*th firm, defined by (5), is

$$\hat{U}_{i} = M_{i}^{*} + \sigma_{*} \{ \phi(-M_{i}^{*}/\sigma_{*}) [1 - \Phi(-M_{i}^{*}/\sigma_{*})]^{-1} \},$$
(13)

where  $M_i^*$  is the random variable which is the counterpart of the mean  $\mu_i^*$ , defined by (9), that is

$$M_i^* \equiv \left(-\sigma^2 \overline{E}_i + T^{-1} \mu \sigma_V^2\right) \left(\sigma^2 + T^{-1} \sigma_V^2\right)^{-1},\tag{14}$$

where

$$\overline{E}_i \equiv T^{-1} \sum_{t=1}^T E_{it}.$$

The predictor  $\hat{U}_i$  is the minimum squared error predictor of  $U_i$ , given  $E_i$ , because it is the conditional expectation of  $U_i$ , given  $E_i$ . Further, it can be shown that  $\hat{U}_i$  is unbiased for  $U_i$  in the sense that  $E(\hat{U}_i) = E(U_i)$ . This property follows from the result

$$E\left\{\phi\left(-M_{i}^{*}/\sigma_{*}\right)\left[1-\Phi\left(-M_{i}^{*}/\sigma_{*}\right)\right]^{-1}\right\}$$
$$=\frac{\sigma_{\nu}}{\left(\sigma_{\nu}^{2}+T\sigma^{2}\right)^{1/2}}\left\{\frac{\phi\left(-\mu/\sigma\right)}{1-\Phi\left(-\mu/\sigma\right)}\right\}.$$

Finally, the predictor  $\hat{U}_i$  is consistent for  $U_i$ , as  $T \to \infty$ , because the random variable  $M_i^*$ , defined by (14), converges in probability to  $\operatorname{plim}_{T\to\infty} - \overline{E}_i = U_i$  and the random variable  $\phi(-M_i^*/\sigma_*)[1 - \Phi(-M_i^*/\sigma_*)]^{-1}$  converges in prob-

ability to zero. The latter result follows because  $-M_i^*/\sigma_*$  is expressible as

$$-M_i^*/\sigma_* = \left\{ \frac{\sigma^2 \overline{E}_i - T^{-1} \mu \sigma_{\nu}^2}{\sigma \sigma_{\nu} (\sigma^2 + T^{-1} \sigma_{\nu}^2)^{1/2}} \right\} T^{1/2},$$

which would be negative for T large enough.

Given that the frontier production function (1)-(2) is defined in terms of the logarithm of production, then a predictor for the technical efficiency of the *i*th firm, defined by (7), is

$$\widehat{TE}_{i} = \left\{ \frac{1 - \Phi \left[ \sigma_{*} - (M_{i}^{*} / \sigma_{*}) \right]}{1 - \Phi \left( -M_{i}^{*} / \sigma_{*} \right)} \right\} \exp \left( -M_{i}^{*} + \frac{1}{2} \sigma_{*}^{2} \right).$$
(15)

This predictor is obtained by replacing  $\mu_i^*$  in (12) by  $M_i^*$ , defined by (14). It is the minimum squared error predictor for  $\exp(-U_i)$ , given  $E_i$ , and is consistent as  $T \to \infty$ .

It should be noted that, given the model (1)-(2), in which firm effects (and technical efficiencies) are time-invariant, the consistency of estimators for individual technical efficiencies requires that the number of time periods increases indefinitely. However, such a situation is unlikely to be realistic, because it is obvious that firm effects and technical efficiencies change, given a sufficiently long period of time.

#### 4. Empirical application to the Australian dairy industry

The Australian Dairy Industry is presently structured according to regulations and requirements within the different states. The states of New South Wales and Victoria are foremost in terms of quantities of milk produced and consumed. The market-milk policies within these two states are substantially different. The New South Wales Dairy Council acquires all milk produced in the state, allocates quotas to individual dairy farms and, until recently, required farmers to produce at least 100% of their quotas in each of the thirteen four-weekly periods of the year. Since July 1984, the Dairy Council relaxed its requirement to apply for at least twelve of the thirteen four-weekly periods. In Victoria, the Dairy Industry Authority has been withdrawing farm specific milk quotas since 1977 and introducing a factory-based quota system. In contrast to New South Wales, there are no penalties for Victorian farms varying their production in different months of the year.

Although the Bureau of Agricultural and Resource Economics annually conducts the Australian Dairy Industry Survey, it appears that there has been no significant production-function analysis of such farm-level data. We consider the estimation of frontier production functions for the New South Wales and Victorian dairy industries. We seek to test whether the mean technical efficiencies in the two states are equal and to predict individual technical efficiencies of dairy farms.

In the Australian Dairy Industry Survey, an eligible dairy farm must have at least thirty dairy cows and receive less than twenty percent of its income from stud or milk-vending enterprises. There is no restriction, however, as to how much income may be derived from other sources. The Dairy Industry Survey involves a stratified rotation sample in which approximately ten percent of farms in the sample drop out each year and are replaced by other farms. We consider data for the three financial years, 1978–79, 1979–80 and 1980–81, in which there were no significant droughts to influence production. We consider data for 69 farms from Victoria and 43 from New South Wales.

The frontier production function specified for the dairy industry in a given state is defined by

$$Y_{it} = \beta_0 + \beta_1 x_{it} + \beta_2 x_{2it} + \beta_3 x_{3it} + V_{it} - U_i, \qquad (16)$$

where the subscript i (i = 1, 2, ..., N) refers to the *i*th sample farm and the subscript t (t = 1, 2, 3) refers to the *t*th year; Y denotes the logarithm of the total gross farm returns, including receipts from crops (net of levies, freight and handling charges, etc.), total livestock trading operating gains and receipts from other sources such as dairy produce, wool, etc.;  $x_1$  denotes the logarithm of the value of total farm labor (in work weeks), which includes the operator's on-farm labor, other family labor, partner or sharefarmer's labor and total hired labor;  $x_2$  denotes the logarithm of the value of the capital, which involves the average estimated replacement cost of structures, plant and equipment, depreciated for age.

The variables of the model (16) are expressed in value terms, rather than physical units, because the latter were not available from the survey data. However, the costs and price structures in the two states are similar because of government requirements. The random variables  $V_{it}$  and  $U_i$  in the model (16) are assumed to have the properties specified for the corresponding unobservable random variables in the frontier production function model (1)–(2).

Ordinary least-squares estimators of the elasticity parameters  $\beta_2$ ,  $\beta_3$  and  $\beta_4$ , in the frontier production function (16) are unbiased (conditional on the values of the independent variables). Because the mean of the random variable,  $U_i$ , is positive, then the ordinary least-squares estimator of the intercept parameter is negatively biased. The ordinary least-squares estimates for the intercept, slope and variance parameters for the production functions for New South Wales and Victoria are presented in table 1. Given that the frontier production function (16) is the true model, then the estimated standard deviations for the estimators of the parameters are not the correct ones for the ordinary least squares (O.L.S.) method.

		Variable				Varian	се рагат	Log- likelihood	Mean technical efficiency	
Region		Intercept	ercept Labor Feed Capital $\tilde{\sigma}^2$		γ	μ				
				N.S	W. (N =	43)			·	
0.L.S.	$(R^2 = 0.85)$	-0.63 (0.54)	0.142 (0.039)	0.394 (0.027)	0.666 (0.055)	0.088	0.0	0.0	- 24.49	_
M.L.	(µ≠0)	-0.38 (0.18)	0.090 (0.011)	0.3558 (0.0077)	0.724 (0.018)	0.108 (31/10 <sup>6</sup> )	0.472 (0.061)	0.20 (0.22)	- 15.49	0.770 (0.114)
M.L.	$(\mu = 0)$	-0.335 (0.047)	0.0840 (0.0029)	0.3602 (0.0026)	0.7220 (0.0044)	0.185 (15/10 <sup>6</sup> )	3.590 (0.037)	0.0	- 17.47	0.768 (0.014)
				Vict	oria (N =	69)				
0.L.S.	$(R^2 = 0.71)$	0.74 (0.48)	0.058 (0.071)	0.200 (0.027)	0.736 (0.044)	0.120	0.0	0.0	- 72.64	-
M.L.	(µ≠0)	1.268 (0.025)	0.0670 (0.0045)	0.0914 (0.0016)	0.8143 (0.0022)	0.202 (12/10 <sup>6</sup> )	0.770 (0.020)	0.39 (0.29)	- 39.32	0.633 (0.095)
M.L.	$(\mu = 0)$	1.688 (0.010)	0.0260 (0.0018)	0.0853 (62/10 <sup>5</sup> )	0.799 (86/10 <sup>5</sup> )	0.407 (27/10 <sup>6</sup> )	0.887 (0.0099)	0.0	- 43.82	0.657 (0.0012)
				Both s	tates (N =	: 112)		- 4 <sub>1</sub>		
0.L.S.	$(R^2 = 0.77)$	0.13 (0.36)	0.137 (0.037)	0.277 (0.017)	0.700 (0.036)	0.116	0.0	0.0	-113.16	
M.L.	(µ≠0)	1.11 (0.17)	0.035 (0.0^33)	0.1696 (0.0012)	0.8160 (0.0028)	0.137 (2/10 <sup>5</sup> )	0.664 (0.020)	0.85 (4.17)	- 66.45	-
M.L.	$(\mu = 0)$	1.01 (0.009)	0.040 (0.0010)	0.179 (0.0005)	0.785 (0.0009)	0.362 (5/10 <sup>5</sup> )	0.865 (0.008)	0.0	- 85.06	

Parameter	estimates	for	frontier	production	functions	for	the	New	South	Wales	and	Victorian
				dairy	industries	.a						

Table 1

<sup>a</sup>Estimated standard deviations of the estimators are given in parentheses below the parameter estimates.

The ordinary least-squares estimates were directly used as initial cstimates, or used to obtain initial estimates, for the Davidon-Fletcher-Powell method of approximating the maximum likelihood (M.L.) estimates for the parameters of the frontier model (16) for the dairy industries in each state and the two states combined. Initial estimates for the ratio-variance parameter  $\gamma \equiv \sigma^2 (\sigma_V^2 + \sigma^2)^{-1}$  were considered between 0.1 and 0.9. A range of initial estimates for the parameter  $\mu$  were considered and corresponding initial estimates vere obtained for the intercept  $\beta_0$  and the total variance parameter  $\tilde{\sigma}^2 \equiv (\sigma_V^2 + \sigma^2)$ .

The approximate maximum-likelihood estimates for the parameters of the frontier models for New South Wales, Victoria and the two states combined are presented in table 1. The estimates for the standard deviations of the maximum-likelihood estimators for the parameters, obtained by the method of Berndt et al. (1974), are presented in parentheses below the maximum-likelihood estimates. Maximum-likelihood estimates are also presented for the parameters of the frontier production function when the positive random

variable  $U_i$  has half-normal distribution (i.e.,  $\mu = 0$ ). Also presented are estimates of the mean technical efficiencies (8) for the original frontier model (16) and the restricted case involving the half-normal distribution.

A joint test on the significance of the random variable  $U_i$  in the frontier mcdel (16) is obtained from the generalized-likelihood ratio. If the random variable is absent from the model (i.e.,  $\mu = \gamma = 0$ ), then the ordinary leastsquares estimators of the remaining parameters of the production function are maximum-likelihood estimators. Thus, the negative of twice the logarithm of the generalized-likelihood ratio has approximately chi-square distribution with parameter equal to two. The values of the test statistic for New South Wales and Victoria are 18.0 and 66.6, respectively, which are highly significant. We thus conclude that both parameters of the distribution of the random variable  $U_i$  are not zero, and so it is significant for describing the distribution of gross farm returns for dairy farms in each state. Further, if the parameter  $\mu$  has value zero, then twice the negative of the logarithm of the generalized-likelihood ratio for the restricted ( $\mu = 0$ ) and unrestricted ( $\mu \neq 0$ ) frontier models has approximately chi-square distribution with parameter equal to one. The values of this statistic are 3.96 and 9.0 for New South Wales and Victoria, respectively. These values are significant at the five percent level and so we conclude that the restricted frontier model ( $\mu = 0$ ) is not an adequate representation for the dairy industries in New South Wales and Victoria.

We note that asymptotic *t*-tests on the estimated  $\mu$  values do not indicate that  $\mu$  is significantly different from zero at the five percent level. It is evident that the parameter estimates for the generalized frontier function (16) are not as precise as those for the restricted model.

An asymptotic chi-square statistic is also used to test if the parameters of the frontier production functions for New South Wales and Victoria are the same. The regative of twice the logarithm of the generalized-likelihood ratio for this problem has approximately chi-square distribution with parameter equal to seven. The value of the test statistic is 23.28, which is significant at the 0.5% level. We, therefore, believe that the frontier production functions for the two states do not have the same parameters.

Additional empirical analyses were conducted in which the frontier model (16) was specified for three regions within each state. These regions were defined by the Bureau of Agricultural and Resource Economics for drawing stratified samples for the Dairy Industry Survey. The regional frontier models within the given states were not significantly different.

Estimates of the mean technical efficiencies, based on the frontier production function (16), indicate that dairy farms in New South Wales are about 77% technically efficient, whereas those in Victoria have technical efficiency of about 63%. These estimates are significantly different at the 20% level for a one-sided asymptotic *t*-test.

Technical efficiency	New South Wales	Victoria		
Below 0.3	0	2 (2.9%)		
0.3-0.4	0	3 (4.3%)		
0.4-0.5	0	11 (15.9%)		
0.5-0.6	4 (9.3%)	19 (27.5%)		
0.6-0.7	9 (20.9%)	15 (21.7%)		
0.7-0.8	9 (20.9%)	9 (13.1%)		
0.8-0.9	18 (41.9%)	8 (11.6%)		
Over 0.9	3 (7.0%)	2 (2.9%)		
Total	3 (100%)	69 (100%)		

Frequencies and percentages of technical efficiencies within decile ranges for New South Wales and Victorian dairy farms.

Table 2

The elasticity estimates obtained for New South Wales and Victoria are also significantly different. The labor and feed elasticities are larger for New South Wales. This may be due to the significant amount of hand feeding on New South Wales' farms during the winter months in order to avoid incurring penalties for not maintaining monthly quotas set by the New South Wales Dairy Council. The elasticity estimate for capital is significantly greater for Victoria than for New South Wales.

Using the estimated parameter values for the frontier production function (16), predictions were obtained for the technical efficiencies (15) of individual dairy farms in New South Wales and Victoria. The values obtained are summarized by reporting the frequencies (and percentages) of farms within the decile ranges indicated in table 2.<sup>1</sup> For dairy farms in New South Wales, the technical efficiencies ranged from 0.548 to 0.927, whereas for Victorian farms, the range was 0.296 to 0.934. Thus, the technical efficiencies of dairy farms in Victoria are much more variable than in New South Wales and are generally lower. This implies that dairy farms in New South Wales operate closer to their frontier production function than do their Victorian counterparts with respect to their frontier production function. This does not necessarily imply that dairy farms in New South Wales are more economically viable than dairy farms in Victoria. In fact, a recent study by Lembit and Bhati (1987) suggests that Victorian dairy farms are generally more cost-efficient than those in New South Wales. This study compared the cost of milk production in regions of the two states separated by the Murray River. These regions have similar

<sup>&</sup>lt;sup>1</sup>Predictions of firm technical efficiencies should be reported with suitable measures of the precision of the predictors (15). These are not presented in this paper because the frequencies of the predicted technical efficiencies are listed in the different ranges for descriptive purposes.

topography and climate, but the farms in the different states operate under different market-milk policies.

#### 5. Conclusions

Our application of stochastic frontier production functions to the dairy industries in New South Wales and Victoria indicates that the traditional (average) Cobb-Douglas production function is not a suitable model. Given that the generalized frontier model (16) applies, then the half-normal distribution is not an adequate representation for the individual firm effects, which determine technical efficiencies of farms. This concurs with the findings of Stevenson (1980) in an application involving only cross-sectional data for the U.S. Primary Metals Industry. The more general model for describing firm effects in frontier production functions accounts for the situations in which there is high probability of firms not being in the neighborhood of full technical efficiency. This is not the case for the half-normal and exponential distributions. However, it is obvious that further research is required on the modelling of technical efficiencies of firms over time for different industries.

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