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PRODUCTION FRONTIERS WITH CROSS-SECTIONAL AND TIME-SERIES VARIATION IN EFFICIENCY LEVELS*

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In this paper we consider the efficient instrumental variables estimation of a panel data model with heterogeneity in slopes as well as intercepts. Using a panel of U.S. airlines, we apply our methodology to a frontier production function with cross-sectional and temporal variation in levels of technical efficiency. Our approach allows us to estimate time-varying efficiency levels for individual firms without invoking strong distributional assumptions for technical inefficiency or random noise. We do so by including in the production function a flexible function of time whose parameterization depends on the firm. We also generalize the results of Hausman and Taylor (1981) to exploit assumptions about the uncorrelatedness of certain exogenous variables with the temporal pattern of the firm's technical inefficiency. Our empirical analysis of the airline industry over two periods of regulation yields believable evidence on the pattern of changes in efficiency across regulatory environments.

1. Introduction

In this paper we consider the efficient instrumental variables estimation of a panel data model in which coefficients in addition to the intercept vary over individuals, and we apply the methodology we develop to a model in which there is cross-sectional and temporal variation in productivity levels (or, equivalently, in levels of technical efficiency), using data on U.S. airlines. We

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therefore extend the current literatures on panel data, productivity measurement, and frontier production functions.

The early literature on stochastic frontier production functions [e.g., Aigner, Lovell, and Schmidt (1977)] assumed the existence of data on a single cross-section of firms, and the separation of technical inefficiency from random noise required strong assumptions about their distributions. More recently, Schmidt and Sickles (1984) considered the case in which panel data are available. In their model only the intercept varied over firms; differences in the intercept were interpreted as differing efficiency levels, with the level of efficiency for each firm assumed to be time-invariant. The Schmidt and Sickles model does not require strong distributional assumptions about technical inefficiency or random noise, nor is the assumption of independence between technical inefficiency and the explanatory variables (inputs) needed. However, the assumption that technical inefficiency is time-invariant is very strong, and depending on the data, may prove unrealistic.

In this paper we seek to relax the assumption that technical inefficiency is time-invariant, but in such a way as to not lose the advantages of panel data. We do so by introducing into the production function a flexible (e.g., quadratic) function of time, with coefficients varying over firms. This function can be thought of as representing productivity growth, at a rate that varies over firms, and it implies that levels of inefficiency for each firm vary over time. This model is similar to the model of Sickles, Good, and Johnson (1986), who considered the measurement of efficiency growth using a profit function which included a flexible function of time, but assumed that efficiency growth was the same for all firms. Our model generalizes their treatment by allowing for cross-sectional variation in productivity growth rates. However, the model still imposes enough structure on the way in which productivity levels change over time that strong distributional assumptions are avoided.

Previous treatments of the linear model with panel data, such as Hausman and Taylor (1981) and Amemiya and MaCurdy (1986), have dealt with the case in which only the intercept varies across individuals (firms). We extend the analysis of Hausman and Taylor to the above model in which there is cross-sectional heterogeneity in slopes as well as (or instead of) intercepts. This case has previously been treated in the random coefficients literature [for example, see Swamy (1971, 1974)], but under the assumption that the variation in coefficients is independent of the regressors; like Hausman and Taylor, we allow some or all of the regressors to be correlated with the cross-sectional variation in coefficients.

The plan of the paper is as follows. Section 2 extends the current panel data literature to a model with heterogeneity in slopes as well as intercept. Section 3 applies our panel data results to the problem of productivity measurement in U.S. airlines, and section 4 gives our empirical results. Section 5 concludes.

2. A panel data model with heterogeneity in slopes and intercept

Our model may be written as

$$y_{it} = X_{it}'\beta + Z_i'\gamma + W_{it}'\delta_i + \varepsilon_{it}, \qquad i=1,\dots,N, \quad t=1,\dots,T, \tag{2.1}$$

where X_{it} is $K \times 1$, Z_i is $J \times 1$, and W_{it} is $L \times 1$, and the parameter vectors β, γ, and δ_i are dimensioned conformably. For the purpose of discussion we can think of the data set being comprised of N individuals (firms) and T time periods per individual. Note that the variables in X and W vary over time, while the variables in Z do not.

The distinguishing feature of our model is that W has coefficients that depend on i. If W just contains a constant, then (2.1) reduces to the standard panel data model in which only the intercept varies across individuals (firms). Let $\delta_i = \delta_0 + u_i$. Then we can write the model as

$$\begin{aligned} y_{it} &= X_{it}'\beta + Z_i'\gamma + W_{it}'\delta_0 + v_{it}, \\ v_{it} &= W_{it}'u_i + \varepsilon_{it}. \end{aligned} \tag{2.2}$$

The u_i are assumed to be iid zero mean random variables with covariance matrix Δ. The disturbances ε_{it} are taken to be iid with a zero mean and constant variance σ^2, and uncorrelated with regressors and u_i.

It is convenient to work with the matrix form of (2.2). This is given by

$$\begin{aligned} y &= X\beta + Z\gamma + W\delta_0 + v, \\ v &= Qu + \varepsilon, \end{aligned} \tag{2.3}$$

where W is $NT \times L$, $Q = \mathrm{diag}(W_i)$, $i = 1,\dots,N$, is $NT \times NL$, and u is $NL \times 1$.

We assume $L \le T$, so that Q is of full column rank. This is not necessary for the identifiability of β. However, it is necessary for estimation of the individual δ_i. Also, if $L > T$, some of the matrices which we must invert would be singular. This is not really a substantive matter, since the projections involved are still well defined, but the algebra would become more complicated. Taking Q to be of full column rank, we denote these projections as follows. Let $P_Q = Q(Q'Q)^{-1}Q'$ be the projection onto the column space of Q and $M_Q = I - P_Q$ be the projection onto the null space of Q.

We derive three different estimators for (2.3), each of which is a straightforward extension of an established procedure for the standard panel data model. The choice between them primarily depends on whether the effects (u_i) are correlated with the explanatory variables (X_{it}, Z_i, and W_{it}).

The first estimator we consider is a generalization of the within estimator from the analysis of covariance. In the standard model, this amounts to transforming the data into deviations from individual means and performing least squares on the transformed data. Similarly, we can transform (2.3) by M_Q and apply least squares. Note that since $M_Q Z = 0$, γ cannot be estimated. The within estimator of β is given by

$$\hat{\beta}_W = (X' M_Q X)^{-1} X' M_Q y. \tag{2.4}$$

The within estimator is an instrumental variables estimator, with instruments M_Q (or, equivalently, $M_Q X$). Its consistency does not depend on assumptions of uncorrelatedness of (X, Z) and (Qu).[1]

Second, we can estimate (2.3) by generalized least squares (GLS). The GLS estimator of $(\beta, \gamma, \delta_0)$ is

$$\left[(X, Z, W)' \Omega^{-1} (X, Z, W)\right]^{-1} (X, Z, W)' \Omega^{-1} y, \tag{2.5}$$

where $\Omega = \mathrm{cov}(v) = \sigma^2 I_{NT} + Q(I_N \otimes \Delta) Q'$. While Ω is a large matrix, it is block-diagonal, with blocks of the form $\sigma^2 I_N + W_i \Delta W_i'$; thus its inversion is practical.

Alternatively, GLS is ordinary least squares (OLS) applied to the transformed equation

$$\Omega^{-1/2} y = \Omega^{-1/2} X\beta + \Omega^{-1/2} Z\gamma + \Omega^{-1/2} W\delta_0 + \Omega^{-1/2} v. \tag{2.6}$$

This transformation was first suggested, for the model with cross-sectional variation only in the intercept, by Fuller and Battese (1973). This expression is not of much actual computational use, however, since $\Omega^{-1/2}$ is harder to calculate than Ω^{-1}; we have

$$\Omega^{-1/2} = \frac{1}{\sigma} M_Q + F, \tag{2.7}$$

where

$$F = Q(Q'Q)^{-1/2}\left[\sigma^2 I_{NL} + (Q'Q)^{1/2}(I_N \otimes \Delta)(Q'Q)^{1/2}\right]^{-1/2} \times (Q'Q)^{-1/2} Q'. \tag{2.8}$$

[This formula follows from a straightforward application of Wansbeek and Kapteyn (1982).]

The consistency of GLS hinges on the uncorrelatedness of (X, Z, W) and Qu. However, GLS allows the estimation of γ, and for fixed T, it is more

[1] More details on the fixed effects treatment of (2.4) can be found in Cornwell (1985).

efficient than the within estimator (2.4). This is exactly the same relationship that exists between GLS and within in the standard model; an explicit proof can be found in Cornwell (1985, sect. 3.3).

Our third estimator is an extension of Hausman and Taylor (1981). Taking an instrumental variables approach, they exploit assumptions about explanatory variables that are uncorrelated with the effects to derive a simple consistent estimator and an asymptotically efficient estimator for the standard panel data model.[2] The extent to which their estimators represent an improvement over the within estimator depends on the number of exogeneity restrictions one is willing to impose. Noting that the within estimator of β always exists and is consistent, Hausman and Taylor use it as a basis of comparison, presenting clear conditions under which their instrumental variables estimators are different.

Following Hausman and Taylor, consider the case in which some of the regressors are correlated with the effects. In particular assume that (X_1, Z_1, W_1) are uncorrelated with the effects, in the sense that $\operatorname{plim}(NT)^{-1}X_1'Qu = 0$, and similarly for Z_1 and W_1, while (X_2, Z_2, W_2) are correlated with the effects. Let the dimensions of X_1, Z_1, W_1, X_2, Z_2, and W_2 be k_1, j_1, l_1, k_2, j_2, and l_2 (with $k_1 + k_2 = K$, $j_1 + j_2 = J$, and $l_1 + l_2 = L$).

A generalization of the Hausman and Taylor simple, consistent estimator is obtained as follows. As in the standard model, we begin with the within estimator, in this case (2.4). The within residuals are

$$(y - X\hat{\beta}_W) = Z\gamma + W\delta_0 + \left[Qu + \varepsilon + X(\beta - \hat{\beta}_W)\right]. \tag{2.9}$$

We transform (2.9) by premultiplying by $\Omega^{-1/2}$:

$$\begin{aligned}\Omega^{-1/2}(y - X\hat{\beta}_W) &= \Omega^{-1/2}Z\gamma + \Omega^{-1/2}W\delta_0 \\ &\quad + \Omega^{-1/2}\left[Qu + \varepsilon + X(\beta - \hat{\beta}_W)\right]. \end{aligned}\tag{2.10}$$

The simple consistent estimator is then defined as instrumental variables of (2.10), using as instruments

$$B^* = \Omega^{-1/2}B = \Omega^{-1/2}(X_1, Z_1, W_1). \tag{2.11}$$

Note that B is transformed by $\Omega^{-1/2}$. Following White (1984, pp. 95–99) the use of untransformed instruments is clearly suboptimal, if we assume 'reduced form' equations for (Z_2, W_2) which are linear in (X_1, Z_1, W_1). This

[2]Amemiya and MaCurdy (1986) introduce an alternative instrumental variables estimator that, under stronger assumptions, is more efficient than the Hausman and Taylor estimator. For a clear exposition of the relationship between the two estimators, see Breusch, Mizon, and Schmidt (1989).

yields the estimator

$$\begin{bmatrix} \hat{\gamma}_W \\ \hat{\delta}_{0W} \end{bmatrix} = \left[(Z,W)'\Omega^{-1/2}P_{B^*}\Omega^{-1/2}(Z,W)\right]^{-1} \times (Z,W)'\Omega^{-1/2}P_{B^*}\Omega^{-1/2}(y - X\hat{\beta}_W). \tag{2.12}$$

The estimator will exist if we have enough instruments, i.e., if the order condition $k_1 + j_1 + l_1 \geq J + L$, or equivalently $k_1 \geq j_2 + l_2$, is satisfied. The corresponding rank condition is that the matrix to be inverted in (2.12) be (asymptotically) of full rank. If it holds, the estimator will be consistent.

To define our efficient instrumental variables estimator, we estimate (2.6) by instrumental variables, using as instruments

$$A^* = \Omega^{-1/2}A = \Omega^{-1/2}(M_Q, X_1, Z_1, W_1). \tag{2.13}$$

Letting $G = (X, Z, W)$, this yields

$$\begin{bmatrix} \tilde{\beta}^* \\ \tilde{\gamma}^* \\ \tilde{\delta}_0^* \end{bmatrix} = \left(G'\Omega^{-1/2}P_{A^*}\Omega^{-1/2}G\right)^{-1}G'\Omega^{-1/2}P_{A^*}\Omega^{-1/2}y. \tag{2.14}$$

Conditions for the existence of (2.14), as well as the relationship between the efficient estimates (2.14) and the simple consistent estimates (2.12) can be summarized as follows. If $k_1 < j_2 + l_2$, $\tilde{\beta}^* = \hat{\beta}_W$ and $(\tilde{\gamma}^*, \tilde{\delta}_0^*)$ does not exist. If $k_1 = j_2 + l_2$, $\tilde{\beta}^* = \hat{\beta}_W$ and $(\tilde{\gamma}^*, \tilde{\delta}_0^*) = (\hat{\gamma}_W, \hat{\delta}_{0W})$, where $(\hat{\gamma}_W, \hat{\delta}_{0W})$ is defined in (2.12). And, if $k_1 > j_2 + l_2$, $(\tilde{\beta}^*, \tilde{\gamma}^*, \tilde{\delta}_0^*) \neq (\hat{\beta}_W, \hat{\gamma}_W, \hat{\delta}_{0W})$ with the former being more efficient than the latter. These results are directly analogous to the results for the standard model given by Hausman and Taylor. See Cornwell (1985, ch. 4, app. A) for proofs of these results.

A remaining detail is the consistent (as $N \to \infty$) estimation of σ^2 and Δ, the unknown parameters in Ω. If SSE_w is the unexplained sum of squares in the within regression, $\hat{\sigma}^2 = SSE_w/N(T-L)$ is a consistent estimate of σ^2. To estimate Δ, let e_i be the IV residuals for individual i (e.g., from the simple consistent IV estimator) and define

$$\hat{\Delta} = \frac{1}{N}\sum_{i=1}^{N}\left[(W_i'W_i)^{-1}W_i'e_ie_i'W_i(W_i'W_i)^{-1} - \hat{\sigma}^2(W_i'W_i)^{-1}\right]. \tag{2.15}$$

A direct calculation reveals that this estimator is consistent [Cornwell (1985, ch. 4, app. B)].

3. A frontiers model with time-varying inefficiency

Schmidt and Sickles (1984) consider the estimation of a stochastic frontier production function with panel data, using the model

$$y_{it} = \alpha + X_{it}'\beta + v_{it} - u_i, \tag{3.1}$$

where y = output, X = inputs, v = statistical noise, and $u > 0$ is a firm effect representing technical inefficiency. This model can obviously be put in the form

$$y_{it} = \alpha_i + X_{it}'\beta + v_{it}, \tag{3.2}$$

where $\alpha_i = \alpha - u_i$. The model (3.2) is of the standard form found in the panel data literature, and β can be estimated by standard methods such as 'within', GLS, or the Hausman and Taylor instrumental variables estimator. It can also be estimated by MLE, assuming a particular distribution for the one-sided error u_i in (3.1). Schmidt and Sickles apply (3.2) to a panel of airlines for the period 1970.I–1977.IV (the period prior to deregulation), assuming a Cobb–Douglas technology. Results from the use of 'within', GLS, and MLE (assuming a half-normal distribution for the firm effects) are compared, and a Hausman–Wu specification error test is carried out to test the null hypothesis that firm-specific effects are uncorrelated with the regressors.

The great benefit of panel data is that one can choose whether to assume particular distributions of v and u, or whether to assume that technical inefficiency is uncorrelated with the inputs, and that therefore these assumptions are testable. However, these benefits come at the cost of the assumption that the firm effects are constant over time. This is a very strong assumption, and probably would be unrealistic in many potential applications. In terms of the Schmidt and Sickles application, as the airline industry moved into the deregulatory transition and beyond, the potential for unstable productivity patterns (reflected in the firm effects) should be clear. Firms within the industry would be expected to respond differently to the new regulatory environment. Although this issue has been dealt with in part by Sickles, Good, and Johnson (1986), the model introduced therein was highly parameterized and required maximum likelihood on a highly nonlinear model. The model we propose here is more parsimoniously parameterized and can be estimated in straightforward ways.

In order to relax the assumption that the firm effects are time-invariant, but in such a way that the advantages of panel data are preserved, we will replace the firm effect (α_i) in (3.2) by a flexibly parameterized function of time, with parameters that vary over firms. The functional form chosen in this paper is a quadratic:

$$\alpha_{it} = \theta_{i1} + \theta_{i2}t + \theta_{i3}t^2. \tag{3.3}$$

Since (3.3) is linear in the elements of θ_{ij} ($j = 1,2,3$), we have exactly the type of model considered in section 2.

In terms of the notation of section 2, we have $W_{it}' = [1, t, t^2]$, $\delta_i' = [\theta_{i1}, \theta_{i2}, \theta_{i3}]$, and with this notation the model (3.2) can be written

$$y_{it} = X_{it}'\beta + W_{it}'\delta_i + v_{it}. \tag{3.4}$$

Clearly the specification (3.3) implies that output levels vary both over firms and over time. Efficiency measurement focuses on the cross-sectional variation, and the model allows efficiency levels to vary over time. Conversely, the measurement of productivity growth focuses on the temporal variation, and the model allows the rate of productivity growth to vary over firms.

Time-varying firm productivity and efficiency levels and rates of productivity growth can be derived from the residuals based on the within, GLS, and efficient instrumental variables estimators presented in section 2.[3] In Schmidt and Sickles (1984), using the model (3.1), the residuals $(y_{it} - X_{it}'\hat{\beta})$ are an estimate of $(v_{it} - u_i)$, and the firm effect (for a given firm) is estimated by averaging its residuals over time. Specifically, the estimate of α_i is

$$\hat{\alpha}_i = \bar{y}_i - \bar{x}_i\hat{\beta}. \tag{3.5}$$

This estimate is consistent as $T \to \infty$. The analogous procedure for the present model is to estimate δ_i by regressing the residuals $(y_{it} - X_{it}'\hat{\beta})$ for firm i on W_{it}; that is, on a constant, time and time-squared. The fitted values from this regression provide an estimate of α_{it} in (3.3) that is consistent (for all i and t) as $T \to \infty$. Finally, in Schmidt and Sickles the frontier intercept α and the firm-specific level of inefficiency for firm i are estimated, respectively, as

$$\hat{\alpha} = \max_j(\hat{\alpha}_j) \quad \text{and} \quad \hat{u}_i = \hat{\alpha} - \hat{\alpha}_i. \tag{3.6}$$

The analogous procedure here is to estimate the frontier intercept at time t and the firm-specific level of technical inefficiency of firm i at time t as

[3]For a discussion of maximum-likelihood estimators for stochastic panel frontiers which treat time-varying inefficiency see Kumbhaker (1990).

follows:

$$\hat{\alpha}_t = \max_j(\hat{\alpha}_{jt}) \quad \text{and} \quad \hat{u}_{it} = \alpha_t - \hat{\alpha}_{it}. \tag{3.7}$$

4. Empirical results

Our data are on U.S airlines over the time period 1970.I–1981.IV, so that $T = 48$. The data follow certificated carriers that existed throughout the study period and that accounted for over 80% of domestic air traffic. Information on output and input prices and quantities was obtained from over 250 accounts from the CAB Form-41. These accounts were aggregated into the four broad input measures of capital, labor, energy, and materials; one output measure, available ton miles; and two output attributes, average stage length (thousands of miles) and service quality. Service quality is based on the number of complaints received by the CAB's Office of Consumer Affairs and is normalized by the number of passenger enplanements for that quarter. The output and input quantities and prices are constructed as Tornqvist indices. We examined the following airlines: American, Allegheny, Delta, Eastern, North Central, Ozark, Piedmont, and United so that $N = 8$. We control for seasonal factors with three dummy variables (with fall the omitted category), and condition on two service attributes, average stage length and quality. For a further discussion of data construction see Sickles (1985) and Sickles, Good, and Johnson (1986). The functional form that we use for (3.1) is a special case of the transcendental logarithmic function [Christensen, Jorgenson, and Lau (1973)]. We assume that the average technology is given by a first-order approximation in the logarithms of input quantities and a second-order approximation in the logarithms of output attributes.[4] In addition, we make the assumptions that input quantities and output characteristics are separable in production, that productivity levels and growths are disembodied, and that seasonal factors are neutral. This reduces the possible number of unrestricted parameters from 66 to 15, a manageable number given the time-series nature of our data, the typical collinearity problems associated with data of this sort, and the use of no additional restrictions embodied in the first-order conditions for output maximization, cost mini-

[4] We attempted to include second-order terms for the inputs, but the almost perfect collinearity in the moment matrix prevented us from obtaining unique parameter estimates. Within results using the generalized inverse gave us an F-statistic of 8.75 for the test of the joint insignificance of the second-order effects of the logarithms of input quantities. The joint insignificance of these parameters is thus not rejected at reasonable significance levels. Instead of dealing with the problem by imposing more structure, e.g., adding optimizing assumptions in the form of first-order conditions to increase the degrees of freedom, we decided let the data and its limitations speak. We simply cannot identify second-order input effects using our data set and largely (or completely) the within variation in variables.

Table 1
Summary statistics (48 quarters, 8 airlines).

Variable	Mean	Standard deviation
ln Q	19.04	1.38
ln K	16.84	1.11
ln L	17.54	1.15
ln E	16.10	1.27
ln M	16.91	1.14
ln stage length	−1.08	0.65
ln quality	−3.36	0.55
(ln stage length)^2	1.59	1.45
(ln quality)^2	11.57	3.79
(ln stage length)* ln quality	3.60	2.18

mization, or profit maximization. The average production technology under consideration is therefore:

$$\ln Q = \ln \alpha_0 + \alpha_k \ln K + \alpha_L \ln L + \alpha_E \ln E + \alpha_M \ln M + \sum_i \delta_i \,\text{Season}_i + \sum_i \gamma_i \ln \text{Attribute}_i + \sum_{i \le j}\sum \gamma_{ij} \ln \text{Attribute}_i \ln \text{Attribute}_j, \tag{4.1}$$

where Q is available ton miles, K, L, E, M are capital, labor, energy, and material input quantities, the seasons are indexed from winter through summer, and where the attributes are average stage length and our service quality index. Summary statistics for the variables in (4.1) are given in table 1.

Estimation results are given in tables 2 and 3. Table 2 displays benchmark GLS and within estimates that are comparable to those given in Schmidt and Sickles (1984) in that only the intercept is allowed to vary across firms. Productivity, however, is allowed to vary over the period. The results of GLS and within are comparable, with energy having the largest output elasticity, followed by labor, materials, and capital. Returns to scale are not significantly different from unity for both estimates at the 95% level, and annual productivity growth is about 1.5% in the median period, 1975.I. The $\bar{R}^2$ for both sets of results is above 0.999. Table 3 presents the within, GLS, and efficient instrumental variables estimates given in (2.4), (2.5), and (2.14). Consider first the GLS and within estimates. The output elasticities do change somewhat across estimation procedures (GLS versus within) as well as across specifications (table 2 versus table 3). The within estimated capital elasticity in table 3 is considerably higher than either estimate in table 2, while the within estimated materials elasticity is considerably lower. Returns to scale are still insignificantly different from unity at the 95% level.

Table 2
Heterogeneity in intercept only.

Variable	GLS Estimate	GLS S.E.	Within Estimate	Within S.E.
ln K	0.183	0.027	0.169	0.027
ln L	0.242	0.030	0.243	0.030
ln E	0.502	0.025	0.500	0.025
ln M	0.203	0.028	0.203	0.028
Winter	0.00198	0.0064	0.00151	0.0060
Spring	0.0223	0.0066	0.0229	0.0062
Summer	0.0284	0.0066	0.0303	0.0062
ln stage length	0.221	0.054	0.101	0.076
ln quality	0.0073	0.041	0.0122	0.040
(ln stage length)^2	0.0434	0.016	0.0103	0.0213
(ln quality)^2	−0.00370	0.0058	−0.00355	0.0058
ln stage length* ln quality	0.0251	0.0081	0.0261	0.0081
Intercept	0.0205	0.290	—	—
Time	0.000591	0.00084	0.0000743	0.00083
Time^2	0.000065	0.000017	0.0000875	0.00000191
σ_u^2	0.00180		—	
σ^2	0.00166		0.00169	

The consistency of the GLS estimates depends on the effects being uncorrelated with all of the explanatory variables. As explained in Schmidt and Sickles (1984), this assumption can be tested using a Hausman–Wu test based on the significance of the differences between the GLS and within estimates. This test statistic equals 17.2. Its asymptotic distribution is chi-squared with 12 degrees of freedom, and a value of 17.2 is significant only at about the 0.15 level. Thus there is some evidence against the exogeneity assumptions underlying the GLS estimator, but it is not significant at usual confidence levels such as 0.05, although this may reflect the low power of the test against nonlocal alternatives.

Despite the insignificance of the evidence against the GLS estimator's exogeneity assumptions, it is reasonable to ask if there is a subset of the explanatory variables for which uncorrelatedness with the effects is more strongly supported by the data. If so, we can impose these uncorrelatedness assumptions using the efficient instrumental variables estimator of section 2. For this purpose we will assume that the seasonal dummy variables and the intercept and time trend variables are uncorrelated with the effects, while the output attribute variables will be treated as correlated with the effects. Correlation patterns between the effects and the input variables were harder to assign *a priori*, but we decided to treat capital and energy as correlated with the effects, and labor and materials as uncorrelated with the effects. We did this for several reasons. The labor input index is based on headcount

Table 3

Heterogeneity in intercept, time, time^2.

Variable	GLS Estimate	GLS S.E.	Within Estimate	Within S.E.	EffIV Estimate	EffIV S.E.
ln K	0.193	0.0303	0.233	0.0321	0.221	0.0276
ln L	0.317	0.0326	0.300	0.0273	0.300	0.0270
ln E	0.466	0.0286	0.498	0.0259	0.494	0.0245
ln M	0.147	0.0300	0.139	0.0249	0.137	0.0242
Winter	0.00189	0.00613	0.00280	0.00491	0.00218	0.00484
Spring	0.0202	0.00629	0.0207	0.00509	0.0211	0.00495
Summer	0.0264	0.00625	0.0252	0.00515	0.0263	0.00496
ln stage length	0.0780	0.0666	−0.0608	0.0902	−0.102	0.0788
ln quality	−0.0383	0.0429	−0.0237	0.0349	−0.0172	0.0342
(ln stage length)^2	−0.0247	0.0255	−0.0874	0.0365	−0.105	0.0335
(ln quality)^2	−0.00714	0.00628	−0.00464	0.00516	−0.00384	0.00505
ln stage length* ln quality	0.0110	0.00883	0.00941	0.00724	0.00992	0.00705
Intercept	−0.0404	0.396	—	—	−0.407	0.406
Time	0.00104	0.00171	—	—	0.000224	0.000214
Time^2	0.0000464	0.0000314	—	—	0.0000465	0.0000372
	0.00407			0.0179		
Δ	−0.000211	0.0000180		−0.000561	0.0000328	
	0.264×10^{-5}	-0.291×10^{-6}	0.552×10^{-8}	0.448×10^{-5}	-0.474×10^{-6}	0.90×10^{-8}
σ^2	0.00166			0.00100		

data. Since adjustment costs for numbers of employees are typically much higher than for hours (which are not measured in the CAB Form-41), any short-run (quarterly) firm shock will likely result in reduced hours or overtime, not in numbers of employees [Schultze (1985) and Shapiro (1986)]. Furthermore, since union contracts cover approximately 70% of the employees in our sample airlines, rational expectations would suggest that any information available to the contracting parties when the contract was made would have been conditioned on, and therefore any unforeseen firm-specific supply shifts would be orthogonal to employment variation while the contract was in force [Sargent (1978)]. The other input which is assumed to be uncorrelated with firm effects is the materials index. This is a residual category, roughly 70% of which is for professional services contracted outside the firm. These include advertising, charter travel bookings, unplanned maintenance of firm's flight equipment by another carrier, and catering services. These data came to us in expenditure form and a Tornqvist index was constructed using a variety of price deflators such as the McCann Erickson Advertising index, the producer price index for miscellaneous business services, and the producer price index for processed foods. The aggregate price indices would have no correlation with airline-specific productivity changes unless firms had a substantial degree of monopsony power in those markets. There is no evidence that this is the case. Whatever weak correlation might have existed between the materials expenditure data and firm productivity effects would be mitigated by the index construction.

The efficient instrumental variables estimates based on these exogeneity assumptions are given in table 3. The coefficient estimates are fairly similar to the within estimates, and there is a slight improvement in the precision of the estimates. Furthermore, the eight uncorrelatedness assumptions that underly the efficient instrumental variables estimator are testable, and the Hausman–Wu statistic (based on differences between the within and efficient instrumental variables estimates) is only 1.08. Thus there is no evidence in the data to make us doubt these exogeneity assumptions.

Table 4 presents the relative efficiency levels derived from our estimates for the carriers at three points in time: 1970.I, 1975.I, 1981.IV. The efficiencies are calculated using the GLS, within, and efficient instrumental variables estimates. As expected, the within and efficient instrumental variables results are quite similar, while GLS efficiency levels and rankings are quite different from within and efficient instrumental variables. In either case there is evidence of considerable change in the efficiency rankings over time; for example, American and United show large improvements in their efficiency rankings from 1970 to 1980.[5]

[5]Productivity levels (%) derived from the within results of table 2 for American, Alleghany, Delta, Eastern, North Central, Ozark, Piedmont, and United are: 96, 80, 100, 89, 78, 82, 81, and 97. Since parameter heterogeneity is allowed for only the constant term, productivity levels are

Table 4

Efficiency levels (%) for selected time periods (1970.I, 1975.I, 1980.IV).

Carrier	GLS	Within	EffIV
American	81, 95, 93	65, 90, 93	72, 93, 94
Alleghany	92, 88, 83	85, 86, 83	86, 86, 80
Delta	92, 99, 99	78, 91, 94	81, 93, 92
Eastern	74, 92, 92	60, 81, 88	64, 84, 87
North Central	86, 100, 88	85, 100, 84	86, 100, 82
Ozark	100, 96, 65	100, 97, 99	100, 96, 93
Piedmont	88, 93, 97	89, 98, 100	90, 97, 94
United	87, 92, 100	66, 84, 100	72, 88, 100

Table 5

Annual productivity growth rates (%) from 1970.I–1981.IV.

Carrier	GLS	Within	EffIV
American	0.45	2.08	1.13
Alleghany	−0.05	−0.42	−1.27
Delta	−0.62	1.08	2.21
Eastern	0.86	2.08	1.24
North Central	−0.38	−0.33	−1.07
Ozark	−3.55	−0.33	−1.30
Piedmont	0.12	0.64	−0.30
United	1.08	2.55	1.60
Output share weighted average	0.44	1.85	1.22

Growth rates in productivity can be calculated by examining the time derivative of the estimate of (3.2). Although these estimates were quite unstable when evaluated period-by-period, we can compare the average values between the first and last period and calculate simple annualized percent rates of growth in total factor productivity (TFP). These calculations are summarized in table 5. Below the rates of total factor productivity growth are the output share weighted averages, which are comparable to the esti-

constant over the sample period, an assumption which is clearly rejected at any reasonable level of significance (F-statistic = 17.06; 0.05, 14,348 = 1.65). Although the constancy of the inefficiencies is rejected, there is still the possibility that productivity rankings may not be affected a great deal. This is not the case, although there does appear to be more concordance between rankings in the later periods. Spearman rank correlations between the productivities based on the standard model and (2.14) are −0.539, −0.476, and 0.428.

mates from the naive model with heterogeneity in the intercept only. We can see that, although magnitudes are not equal, the estimates based on within and efficient instrumental variables are of the same sign (except for Piedmont which is quite small) and roughly the same magnitude. TFP growth rates calculated from the (probably misspecified) GLS estimates are quite different from the within and efficient instrumental variables TFP growth rates and suggest an industry average growth rate of 0.44, versus the 1.22–1.85 implied by the consistent within and efficient instrumental variables estimates.

It is obvious in table 4 that, on average, the firms in our sample became more efficient over time. The average level of efficiency for our eight firms is roughly 82% in 1970.I and grows to almost 95% in 1980 before dropping slightly in 1981. It is important to stress that this increase in efficiency levels is not just a reflection of the fact that there was productivity growth over the sample period. A firm's efficiency level for a given time period is calculated by comparing the firm's output to the frontier level calculated using the production function of the most efficient firm [the one with the highest intercept u_{it} in eq. (3.2)]; see eq. (3.5). Thus the empirical fact that drives an increase in efficiency levels over time is that the firms' productivity levels are becoming more similar over time. It is easy to conjecture that this is due to increasing competitive pressures in the airline industry over the sample period, although in fact most of the increase in average efficiency levels occurred before the formal passage of the air deregulation act in late 1978.

The temporal pattern of changes in efficiency levels displayed in table 4 is of obvious interest. It indicates exactly the kind of detail available in the present model and not available in the simpler model of Schmidt and Sickles (1984).

6. Conclusions

In this paper we have specified a simple model which, in the presence of panel data, allows us to estimate time-varying efficiency levels for individual firms, without making strong distributional assumptions for technical inefficiency or random noise. We do so by including in the production function a flexible function of time, with parameters that differ across firms. We also generalize the earlier econometric results of Hausman and Taylor (1981) to develop an econometric technique that allows us to choose how many explanatory variables we wish to assume to be uncorrelated with the firm's temporal pattern of productivity growth. We have used this model and these estimators to analyze the U.S. airline industry during two periods of regulation and obtained results that are quite intuitive and reasonable, including believable evidence on the pattern of changes in efficiency across regulatory environments.

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