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PRODUCTION FRONTIERS, PANEL DATA, AND TIME-VARYING TECHNICAL INEFFICIENCY

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This paper uses a panel-data framework and models firm-specific technical inefficiency which is allowed to vary over time. The specification is flexible enough to accommodate increasing, decreasing, and time-invariant behavior of technical inefficiency. Time-varying firm- and input-specific allocative inefficiency is also incorporated. The estimation method suggested uses a parametric production function and cost-minimization hypothesis.

1. Introduction

One of the major objectives of studying production and cost frontiers is to estimate economic efficiency of the production units (firms). A cost-minimizing firm may be inefficient due to (i) technical inefficiency and/or (ii) allocative inefficiency. In estimating technical inefficiency using a cross-section of firms, one has to assume technical inefficiency to be random and specific distributional assumptions are required [see Aigner et al. (1977)]. But, if panel data are available one can estimate technical inefficiency without specifying distributional assumptions, provided it is time-invariant [see Schmidt and Sickles (1984), Kumbhakar (1987a)]. However, maximum-likelihood estimators given appropriate distributional assumptions can be more efficient.[1]

The assumption of time-invariant technical inefficiency may not be as innocuous as it appears. Imposing this restriction without formally testing its appropriateness may result in inconsistency of estimators for the parameters of the model as well as for technical and allocative inefficiency. There are two important papers in which the time-invariant assumption of technical and allocative inefficiency has been relaxed. Sickles et al. (1986) modelled allocative inefficiency where firms adjust output supplies and input demands to the wrong price ratios. These price distortions are allowed to vary over time which makes allocative inefficiency time-dependent. However, these are two problems. First, allocative inefficiency is input-specific but not firm-specific.

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[1]See Kumbhakar (1988) for a case where both technical and allocative inefficiency are time-invariant.

Second, the model does not allow for technical inefficiency. The model developed by Cornwall et al. (1990), on the other hand, does allow technical inefficiency to vary over time by specifying it as a quadratic function of time. But, since this model is a single-equation framework, in which only the production function is used, allocative inefficiency is not captured by the model. This may result in inconsistent estimators if the inputs are endogenous. However, one major advantage of both these models is that no 'special' distributional assumptions are needed for technical or allocative inefficiency.

In contrast to these formulations, we present a model that accommodates both technical and allocative inefficiency. Using the cost-minimization framework, we model (i) time-varying technical inefficiency and (ii) allocative inefficiency that varies over time and across firms. The model also accommodates exogenous inputs. The maximum-likelihood method is suggested, based on the usual distributional assumptions on technical and allocative inefficiency. Thus the present model overcomes the limitations of the Sickles et al. (1986) and Cornwell et al. (1990) formulations. The only disadvantage is its dependence on distributional assumptions.

The paper is organized as follows. The model is formulated in section 2. Estimation techniques are discussed in section 3. Section 4 generalizes some results and section 5 summarizes the paper.

2. The model

We start with a somewhat restricted form of the translog production function,[2] namely,

$$y = A\left(\prod_{i=1}^{n} x_i^{\alpha_i}\right)\left(\prod_{j}^{m} z_j^{\beta_j}\right)\exp\left(\sum_i \sum_j \delta_{ij} \ln x_i \ln z_j\right)\mathrm{e}^{u+v}, \qquad (1)$$

where u (≤ 0) contributes to technical inefficiency, v is a white noise random error, x_i ($i = 1,\dots,n$) are variable inputs and z_j ($j = 1,\dots,m$) are exogenous shift variables. We assume that the firm's objective is to minimize cost subject to the stochastic production function (1). A cost-minimizing firm is said to be allocatively inefficient if $f_i/f_1 \neq w_i/w_1$, where f_i is the marginal product of input x_i and w_i is its price. Following Schmidt and Lovell (1979), allocative inefficiency can be modelled as

$$\ln x_1 - \ln x_i - \ln(w_i/w_1) + \ln\left(\alpha_i + \sum_j \delta_{ij} \ln z_j\right) - \ln\left(\alpha_1 + \sum_j \delta_{1j} \ln z_j\right) = \xi_i, \qquad i = 2,\dots,n, \qquad (2)$$

[2]The production function is assumed to satisfy monotonicity and convexity conditions.

where ξ_i represents allocative inefficiency for the input pair $(j,1)$. The parameters of the model outlined in (1) and (2) can be estimated by the maximum-likelihood (ML) method, given suitable distributional assumptions on u, v, and $\xi_2,\ldots,\xi_n$. Once the parameters are estimated, allocative inefficiency $\xi_2,\ldots,\xi_n$ can be estimated from (2). Technical inefficiency, u, can then be estimated from the mean or mode of the conditional distribution of u given $(u+v)$, as proposed by Jondrow et al. (1982). One can also estimate technical efficiency, $\exp(u)$ given $(u+v)$.

The estimates of technical inefficiency are not consistent if the model is estimated using data from a cross-section of firms. This problem can potentially be avoided by using panel data. However, some additional assumptions are required regarding the time behavior of technical and allocative inefficiency. The usual assumptions in the literature are [see Battese and Coelli (1988), Kumbhakar (1987b, 1988), Pitt and Lee (1981), Schmidt (1986), and Schmidt and Sickles (1984)]:

(i) u_{ft} are time-invariant and distributed as i.i.d. $N(0,\sigma_\tau^2)$ truncated at zero from above,
(ii) $\xi_{ft} \sim$ i.i.d. $N(0,\Sigma)$ for all f and t, where $\xi_{ft}=(\xi_{2ft},\ldots,\xi_{nft})'$,
(iii) $v_{ft} \sim$ i.i.d. $N(0,\sigma_v^2)$ for all f and t,
(iv) v_{ft}, u_{ft}, and ξ_{ft} are independent for all f and t,

where f indexes firm $(f=1,2,\ldots,F)$ and t indexes time $(t=1,2,\ldots,T)$.

The most serious of these assumptions is (i), especially if u is assumed to be known to the firm. If so, how can a firm do nothing over time knowing that it is technically inefficient? Empirically it would be advantageous to test whether u is time-invariant. Simply imposing it in the model without testing may cause inconsistent estimators of the parameters, and of technical inefficiency. Assumptions (ii) and (iii) are standard and reasonable. But assumption (iv) may require some justification. It is reasonable to assume that technical and allocative inefficiency (u and ξ) are independent of random statistical error (v), since v is not under the control of any firm while u and ξ are. On the other hand, the assumption of independence of u and ξ is questionable. Schmidt and Lovell (1980) have developed a technique to test this independence assumption. However, the introduction of correlated inefficiencies requires some additional distributional assumptions which may be arbitrary. But, even apart from these, the model becomes very complicated, and so we have retained the independence assumption.

In this paper we consider a formulation of technical inefficiency which is flexible enough to handle different types of time behavior. Time-invariant inefficiency becomes a special case that can be statistically tested by a t-test or by a generalized likelihood-ratio (LR) test.

Let $u_{ft}=\gamma(t)\tau_f$, where $\gamma(t)$ is a well defined function of t and τ_f is time-invariant but varies across firms. We assume τ_f to be random, distributed as i.i.d. $N(0, \sigma_t^2)$ and truncated at zero $\tau_f \leq 0$. There can be a wide range of choices for $\gamma(t)$. We consider the following:

$$\gamma(t) = \left(1 + \exp(bt + ct^2)\right)^{-1}. \tag{3}$$

This particular form of $\gamma(t)$ has the following features:

(i) $\gamma(t) \geq 0$ for all t, which implies $u_{ft} \leq 0$ since $\tau_f \leq 0$.
(ii) $\gamma(t)$ is bounded between $(0, 1)$.
(iii) $\gamma(t)$ can be monotonically increasing (decreasing) or concave (convex) depending on the signs and magnitude of b and c.

Thus (iii) allows the data to determine the time behavior of $\gamma(t)$ and hence u_{ft}, instead of imposing it *a priori*. If $b + ct < 0$ (> 0) for all t, a simpler functional form can serve the purpose. For example,

$$\gamma(t) = (1 + \exp(bt))^{-1} \tag{4}$$

can show either monotonically increasing ($b < 0$) or decreasing ($b > 0$) behavior of technical inefficiency. The time-invariance assumption on u_{ft} is equivalent to $b = 0$, which may be tested with an asymptotic t-test or a LR test. On the other hand, if specification (3) is used, then u_{ft} will be time-invariant if $b = c = 0$, which can be tested by using a LR test.

3. Estimation

Rewriting (1) in logarithmic form and introducing firm and time subscripts yields

$$\begin{aligned}\ln y_{ft} &= \alpha_0 + \sum_i \alpha_i \ln x_{ift} + \sum_j \beta_j \ln z_{jft} + \sum_i \sum_j \delta_{ij} \ln x_{ift} \ln z_{jft} + \theta_{ft} \\ &\equiv \alpha_0 + \alpha' \ln x_{ft} + \beta' \ln z_{ft} + \ln x'_{ft} \Delta \ln z_{ft} + \theta_{ft},\end{aligned} \tag{5}$$

where

$$\theta_{ft} = u_{ft} + v_{ft} = \gamma(t)\tau_f + v_{ft}.$$

It is inappropriate to estimate (5) with Ordinary Least-Squares (OLS) if y is exogenous and x_i endogenous, as is the case in a cost-minimization problem.

On the other hand, if y is endogenous and everything else is exogenous, OLS may seem appropriate. However, in fact, OLS parameter estimates will be biased and inconsistent because of the following.

Since the error term is $\theta_{ft} = \gamma(t)\tau_f + v_{ft}$, its expected value, $\mathrm{E}(\theta_{ft}) = \gamma(t)\mathrm{E}(\tau_f) = \gamma(t)\mu$ is nonzero where μ is the mean of τ_f. Thus omission of $\gamma(t)\mu$ from (5) (which is the case when OLS is used) will cause bias and inconsistency in the coefficients of the variables with which it correlates. However, if $\gamma(t)$ is linear in t and if one of the $\ln z$ variables in the regression equation is time, then omission of $\gamma(t)\mu$ will bias only the coefficients of time and the intercept. This is the generalization of the familiar result that OLS estimates all of the parameters consistently except the intercept in the standard frontier model which corresponds to $\gamma(t) =$ constant. Thus OLS is inapplicable for obtaining consistent parameter estimators of all the parameters especially when technical inefficiency varies over time. However, this inconsistency can be avoided by using the ML method.

The system of equations for the ML method consists of (5) and (2). Denoting $\theta_{ft} = u_{ft} + v_{ft}$, the residual vector can be written as $(\theta_{ft}, \xi_{ft})'$, where $\xi_{ft} = (\xi_{2ft}, \ldots, \xi_{nft})'$. Let $\boldsymbol{\theta}_f = (\theta_{f1}, \ldots, \theta_{fT})'$ and $\boldsymbol{\xi}_f = (\xi_{ft}, \ldots, \xi_{fT})'$. The joint pdf of $\boldsymbol{\theta}_f$, $\boldsymbol{\xi}_f$, and τ_f, $f(\boldsymbol{\theta}_f, \boldsymbol{\xi}_f, \tau_f) = f(\boldsymbol{\theta}_f, \tau_f) \cdot g(\boldsymbol{\xi}_f)$, since $\boldsymbol{\xi}_f$ is independent of $\boldsymbol{\theta}_f$ and τ_f. $g(\boldsymbol{\xi}_f)$ is the pdf of $\boldsymbol{\xi}_f$. Furthermore, $f(\boldsymbol{\theta}_f, \tau_f) = f(v_{f1}, \ldots, v_{fT}, \tau_f) = f(\tau_f) \cdot (\prod_t f(v_{ft})) = f(\tau_f) \prod_t f(\theta_{ft} - \gamma(t)\tau_f)$, since τ_f and $v_{f1}, \ldots, v_{fT}$ are independent and v_{ft} is i.i.d.

Now,

$$
\begin{aligned}
f(\boldsymbol{\theta}_f) &= \int_{-\infty}^{0} f(\boldsymbol{\theta}_f, \tau_f)\,\mathrm{d}\tau_f \\
&= \int_{-\infty}^{0} \prod_t f(\theta_{ft} - \gamma(t)\tau_f) \cdot f(\tau_f)\,\mathrm{d}\tau_f \\
&= \frac{2}{(2\pi)^{(T+1)/2}\sigma_v^T\sigma_\tau} \\
&\quad \cdot \int_{-\infty}^{0} \exp\left\{-\tfrac{1}{2}\left[\sum_t (\theta_{ft} - \gamma(t)\tau_f)^2/\sigma_v^2\right] - \tau_f^2/2\sigma_\tau^2\right\}\mathrm{d}\tau_f \\
&= \frac{2\sigma_* \exp(-a_f^*/2)}{(2\pi)^{T/2}\sigma_v^T\sigma_\tau} \cdot \Phi(-\mu_f^*/\sigma_*),
\end{aligned}
$$

where

$$\sigma_* = \frac{\sigma_v \sigma_\tau}{\left(\sigma_v^2 + \sigma_\tau^2 \sum_t \gamma^2(t)\right)^{1/2}},$$

$$\mu_f^* = \frac{\sum_t \gamma(t)\theta_{ft}}{\sigma_v^2} \cdot \sigma_*^2 = \frac{\sigma_\tau^2 \sum_t \gamma(t)\theta_{ft}}{\sigma_v^2 + \sigma_\tau^2 \sum_t \gamma^2(t)},$$

$$a_f^* = \frac{1}{\sigma_v^2}\left\{ \sum_t \theta_{ft}^2 - \left(\sigma_\tau^2 \left(\sum_t \gamma(t)\theta_{ft}\right)^2\right)\left(\sigma_v^2 + \sigma_\tau^2 \sum_t \gamma^2(t)\right)^{-1}\right\}.$$

The log-likelihood function for a sample of F firms observed over T years is

$$L = \sum_f \ln f(\boldsymbol{\theta}_f) + \sum_f \ln g(\boldsymbol{\xi}_f) + \sum_f \sum_t \ln |J_{ft}|, \tag{6}$$

where

$$|J_{ft}| = \sum_i \alpha_i + \sum_i \sum_j \delta_{ij} \ln z_{jft}$$

is the Jacobian of the transformation from $(\theta_{ft}, \xi_{ft}')'$ to $(\ln x_{1ft}, \ldots, \ln x_{nft})'$ obtained from (5) and (2), and

$$g(\boldsymbol{\xi}_f) = \frac{1}{(2\pi)^{T(n-1)/2} |\Sigma|^{1/2}} \exp\left\{ -\tfrac{1}{2} \sum_t \xi_{ft}' \Sigma^{-1} \xi_{ft} \right\}.$$

Finally, $\boldsymbol{\theta}_f$ and $\boldsymbol{\xi}_f$ in (6) are to be replaced by their observable counterparts from (5) and (2). The ML estimates of $\alpha_0, \alpha_i, \beta_j, \delta_{ij}, b, c, \sigma_v^2, \sigma_\tau^2$ can be obtained by maximizing (6).

3.1. Estimation of τ, u, and ξ

To estimate τ_f we need the pdf of τ_f, given $\boldsymbol{\theta}_f$, $f(\tau_f|\boldsymbol{\theta}_f)$. It can easily be shown that

$$f(\tau_f|\boldsymbol{\theta}_f) = \frac{1}{\sqrt{2\pi}\,\sigma_*} \frac{\exp\left\{ -\frac{1}{2\sigma_*^2}(\tau - \mu_f^*)^2 \right\}}{\Phi(-\mu_f^*/\sigma_*)}, \qquad \tau_f \le 0, \tag{7}$$

which is the pdf of a normal variable truncated at zero. A point estimator of τ_f can be found [see Jondrow et al. (1982) and its extension in Kumbhakar (1987b)] by the mean or the mode of $\tau_f|\boldsymbol{\theta}_f$, viz.,

$$\hat{\tau}_f = \mathrm{E}(\tau_f|\boldsymbol{\theta}_f) = \mu_f^* - \sigma_* \frac{\phi(\mu_f^*/\sigma_*)}{\Phi(-\mu_f^*/\sigma_*)}, \tag{8}$$

or

$$\tilde{\tau}_f = \mathrm{Mode}(\tau_f|\boldsymbol{\theta}_f) = \begin{cases} \mu_f^* & \text{if } \sum_t \theta_{ft}\gamma(t) \le 0, \\ 0 & \text{otherwise}. \end{cases} \tag{9}$$

It can also be shown that the estimators in (8) and (9) are consistent as $T \to \infty$. If technical efficiency is to be estimated, then the best predictor is $\mathrm{E}(\exp(\tau_f \gamma(t)|\boldsymbol{\theta}_f))$.

Once τ_f is estimated either by (8) or (9), u_{ft} for each firm can be estimated from

$$\hat{u}_{ft} = \hat{\gamma}(t)\hat{\tau}_f, \tag{10}$$

where

$$\hat{\gamma}(t) = \left(1 + \exp(\hat{b}t + \hat{c}t^2)\right)^{-1}. \tag{11}$$

Since the ML estimates of b and c, $\hat{b}$ and $\hat{c}$, are consistent, so is $\hat{\gamma}(t)$. Thus $\hat{u}_{ft}$ is a consistent estimator of u_{ft}.

Input- and firm-specific allocative inefficiency, ξ_i can be estimated for every time period from

$$\hat{\xi}_{ift} = \ln x_{1ft} - \ln x_{ift} + \ln w_{1ft} - \ln w_{ift} + \ln\left(\alpha_i + \sum_j \delta_{ij} \ln z_{jft}\right)$$
$$- \ln\left(\alpha_1 + \sum_j \delta_{1j} \ln z_{jft}\right), \qquad i = 2, \ldots, n. \tag{12}$$

3.2. *Costs of technical and allocative inefficiency*

Since inefficiency increases cost, it is of some interest to compute the increase in cost due to technical and allocative inefficiency. To do this, we first derive the conditional input demand functions from (1) and (2). These are[3]

$$\ln x_1 = \frac{1}{l'\kappa}\left[\ln y - \alpha_0 - q'\kappa + \psi'\kappa - (\tau + v)\right],$$

$$\ln x_i = \ln x_1 + \ln m_i - \ln(w_i/w_1) - \xi_i, \tag{13}$$

where

$$\kappa = \alpha + \Delta \ln z, \qquad l_{nx1} = (1,\dots,1)', \qquad \psi = (0, \xi_2, \dots, \xi_n)',$$

$$q = \{0, \ln m_2 - \ln(w_2/w_1), \dots, \ln m_n - \ln(w_n/w_1)\}',$$

$$m_i = \left(\alpha_i + \sum_j \delta_{ij} \ln z_j\right) \Big/ \left(\alpha_1 + \sum_j \delta_{1j} \ln z_j\right), \qquad i = 2, \dots, n.$$

From the preceding input demand functions we can derive the cost function

$$\ln(C/w_1) = \frac{1}{l'\kappa}[\ln y - \alpha_0 - q'\kappa] + \ln\left[1 + \sum_i m_i \mathrm{e}^{-\xi_i}\right]$$

$$+ \frac{1}{l'\kappa}[\psi'\kappa - \tau - v]. \tag{14}$$

The cost frontier is obtained by setting $\tau = \xi_i = 0$, namely,

$$\ln(C^*/w_1) = \frac{1}{l'\kappa}[\ln y - \alpha_0 - q'\kappa] + \ln\left(1 + \sum_i m_i\right) - v/(l'\kappa). \tag{15}$$

Increase in cost due to only allocative inefficiency, $\ln C_A$, can therefore be calculated from

$$\ln C_A = \ln(C/w_1) - \ln(C/w_1 \text{ with } \xi_i = 0)$$

$$= \frac{\psi'\kappa}{l'\kappa} + \ln\left[\frac{e'\kappa}{l'\kappa}\right], \tag{16}$$

where

$$e = (1, e^{-\xi_2}, \dots, e^{-\xi_n})'.$$

[3]For notational convenience we have dropped the firm and time subscripts f and t in this subsection.

Similarly, the increase in cost due only to technical inefficiency, $\ln C_{\tau}$, can be calculated from

$$\ln C_{\tau} = \ln(C/w_1) - \ln(C/w_1 \text{ with } \tau = 0) = -\tau/(l'\kappa). \tag{17}$$

It is to be noted that $\ln C_A$ and $\ln C_{\tau}$ are firm-specific and vary over time.

4. Some generalizations

The basic model of this paper can be modified to accommodate a wide class of situations. First, the methodology can accommodate any parametric production function. For example, it is quite straightforward to derive the system of equations similar to (5) and (2) from a translog production function.[4] The log-likelihood function, as well as estimates of technical and allocative inefficiency can also be similarly derived. However, one cannot derive estimates of increase in cost due to technical and allocative inefficiency since the conditional input demand functions cannot be analytically derived for the translog production function.

Second, if output is endogenous and inputs are all exogenous, one can focus only on technical inefficiency and use a single-equation method. Any parametric production function can be used. The likelihood function for a single firm is simply the pdf of $\boldsymbol{\theta}_f, f(\boldsymbol{\theta}_f)$. Technical inefficiency can be estimated by using the formulas in (8) or (9).

On the other hand, if the objective is to estimate the cost of technical and allocative inefficiency and the underlying behavioral assumption is cost minimization, one can start with the cost function. Flexible functional forms, such as translog, Generalized Leontief, Symmetric Generalized McFadden [see Diewert and Wales (1987)], etc., can be accommodated.

Third, the distributional assumptions on technical inefficiency may be restrictive. This is especially true when panel data are used, since the principal advantage of panel data in the estimation of frontier functions is that distributional assumptions which are necessary in a cross-section can be avoided. Distributional assumptions can be avoided by considering a fixed effects treatment of technical inefficiency. For example, consider

$$u_{ft} = \gamma_{0f} + \gamma_{1f}t + \gamma_{2f}t^2, \tag{18}$$

where γ_0, γ_1, and γ_2 are firm-specific [Cornwell et al. (1990)]. Substituting

[4]See Kumbhakar (1989) for a discussion on the estimation of technical and allocative inefficiencies under the behavioral assumption of profit maximization. The analysis can be extended to a panel data model where technical inefficiency varies over time.

(18) in (5) yields

$$\ln y_{ft} = \sum_i \alpha_i \ln x_{ift} + \sum_j \beta_j \ln z_{jft} + \sum_i \sum_j \delta_{ij} \ln x_{jft} \ln z_{jft}$$

$$+ \delta_{0f} + \delta_{1f} t + \gamma_{2f} t^2 + v_{ft}, \tag{19}$$

where

$$\delta_{0f} = \gamma_{0f} + \alpha_0 \quad \text{and} \quad \delta_{1f} = \gamma_{1f} + \beta_t.$$

β_t is the coefficient of time[5] (which represents exogenous technical progress). The system of equations in (19) and (2) can be used to estimate all of the parameters. Finally, technical inefficiency relative to the most efficient firm (RTI) can be estimated from

$$RTI_{ft} = \left(\hat{\delta}_{0f} + \hat{\delta}_{1f} t + \hat{\delta}_{2f} t^2\right) - \max_f \left[\hat{\delta}_{0f} + \hat{\delta}_{1f} t + \hat{\gamma}_{2f} t^2\right], \tag{20}$$

$$f = 1, \ldots, F, \quad t = 1, \ldots, T.$$

Fourth, allocative inefficiency, ξ_{ft}, is assumed to be independent over time with mean zero. This assumption can be relaxed somewhat by introducing a nonzero mean so that part of allocative inefficiency is constant over time [see Kumbhakar (1988) and Schmidt (1988)].

5. Summary

This paper presents an estimable model in which technical inefficiency is allowed to vary across firms and over time. The specification used is quite flexible. It can accommodate increasing, decreasing, and time-invariant behavior of technical inefficiency. And, based on the assumption of cost minimization, time-varying firm- and input-specific allocative inefficiency is also incorporated. The ML estimation method, based on a parametric production function, is developed to estimate the parameters. Estimates of technical and allocative inefficiency based on the ML parameter estimates are also suggested. Finally, formulas for calculating costs of technical and allocative inefficiency are derived.

The results of this paper are applicable to a wide class of models. Any flexible production/cost function can be accommodated in the framework, with inputs treated as endogenous or exogenous. Similarly, technical inefficiency can be specified as fixed effects without the necessity of any distribu-

[5]It is assumed that one of the $\ln z$ variables is time which is separated out.

tional assumptions, and allocative inefficiency can have nonzero mean constant over time.

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