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# THE MEASUREMENT AND SOURCES OF TECHNICAL INEFFICIENCY IN THE INDONESIAN WEAVING INDUSTRY*

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Production function models are estimated with a time series of cross-section data on Indonesian weaving establishments. The sources of technical inefficiency are investigated. Three firm attributes are identified as being potentially related to firm efficiency. They are firm ownership, age and size. The importance of these attributes as sources of inefficiency in the Indonesian weaving industry is investigated and the implications of the findings discussed.

## 1. Introduction

The relative efficiency of manufacturing firms in developing countries has been a topic of considerable interest in development literature. For example, opponents and proponents of foreign-investment in LDC manufacturing have made conflicting assertions regarding the relative efficiency of foreign firms compared to private domestic firms. Similarly, conflicting claims have been made concerning the efficiency of firms using capital-intensive techniques similar to those used in developed countries relative to firms using labor-intensive techniques. Often, what is meant by efficiency is not clearly stated and attempts at its measurement make use of output–input ratios, particularly labor productivity, which are without theoretical foundation. In order to investigate the sources of inefficiency, it is first necessary that efficiency be measured in a manner consistent with its theoretical definition.

In this paper, frontier production function models are proposed and estimated with a time series of cross-section data on Indonesian weaving establishments. The appropriateness of alternative model formulations are statistically tested. An investigation of the sources of inefficiency identifies three firm attributes as being potentially related to efficiency. They are firm

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ownership, age and size. The importance of these attributes as sources of inefficiency in the Indonesian weaving industry is investigated by explicitly including them in the proposed model and also through traditional analysis of covariance. The policy implications of the findings are then discussed.

## 2. Estimating the efficiency of production

### 2.1. Approaches to measuring efficiency

Technically efficient production is defined as the maximum quantity of output attainable from given inputs. Knowledge of the production frontier, defined as the locus of technically efficient input–output combinations, and the actual input–output combinations of firms is sufficient information for measuring efficiency. A major difficulty is estimating the production frontier. Typically, empirical production functions are 'average' rather than frontier functions, and thus unable to provide information on efficiency, because they attribute differences from the estimated function to symmetric random disturbances. Attempts to estimate frontier production function began with the pioneering work of Farrell (1957) and subsequently, Aigner and Chu (1968), Afriat (1972) and Richmond (1974). They estimated the frontier using linear and quadratic programming techniques. There are several disadvantages to their approach. The most important problem is that it does not allow for random shocks in the production process which are outside the firms control. As a consequence, a few extreme measured observations determine the frontier and exaggerate the maximum possible output given inputs.

Recognizing this problem, Timmer (1971) eliminated a certain percentage of the total observations. Such a selection procedure, however, is not based in statistical theory and the number of observations eliminated is arbitrary. Recently, Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977) handled this problem with a more satisfactory conceptual basis by explicitly including an efficiency component in the error term of the estimated production function. However with the exceptions of Meeusen and van den Broeck (1977) and Lee and Tyler (1978), empirical investigations utilizing these new techniques is limited and not entirely satisfactory.

In this paper, models which are appropriate for analyzing panel data are considered and generalized [cf. Kmenta (1971, ch. 12)]. One specification is a form of the random effect variance components model. Estimation of average production functions using variance components models (fixed effect as well as random effect models) was the topic of Nerlove (1965), Mundlak (1961), Hoch (1962), Timmer (1971) and others. Our model generalizes these models to incorporate the stochastic frontier production functions approach originated in Aigner et al. (1977). Although this specification implicitly assumes that firm inefficiency is time invariant, it has the advantage of providing a measure of average efficiency. The other specification permits firm efficiency to vary over time and is related to Zellner's (1962) seemingly unrelated regressions. The models are then applied to pooled micro data obtained from individual Indonesian weaving firms. The latter model contains the former model as a special case which is empirically testable.

Pooled data is preferred in this analysis for at least four reasons. First, observing firms over a number of years permits us to test for structural change in the production function. Second, it is not possible to estimate the efficiency of individual firms from a single cross-section. Third, the use of pooled data permits the comparison of our approach to the traditional analysis of covariance approach. Fourth, it permits us to investigate whether the inefficiency of firms is time variant or time invariant, and if it is time variant, whether or not it varies randomly. These provide information about the behavior of firms over time which cannot be revealed from cross-sectional data.

### *2.2. Model specification*

Consider the production function model with multiplicative disturbances

$$z = f(x, \beta)\,e^{\varepsilon}, \tag{1}$$

where $x$ is a $1 \times K$ row vector of inputs, $f(x, \beta)$ is the theoretical maximum output, $z$ is the observed output and $e^{\varepsilon}$ is the stochastic error term. The stochastic frontier specification of Aigner et al. (1977) and Meeusen and van den Broeck (1977) differs from previous studies in that the error, $\varepsilon$, is composed of two different types of disturbances

$$\varepsilon = u + v, \tag{2}$$

where $u$ is one-sided distributed, $u \leqq 0$, which represents technical inefficiency and $v$ is a stochastic variable which represents uncontrolled random shocks in the production process. The non-positive disturbance $u$ reflects the fact that output must lie on or below its frontier $f(x, \beta)e^{v}$, since $e^{u}$ has a value between zero and one. The frontier $f(x, \beta)e^{v}$ is stochastic as $v$ consists of random factors beyond the firms control.

To simplify the estimation, a log-linear model will be considered. After a logarithmic transformation, (1) is simply

$$y = x\beta + \varepsilon, \tag{3}$$

where $y = \ln z$. To generalize the model (3) to handle cross section and time series data, we consider the following variance components model:

$$y_{it} = x_{it}\beta + u_{it} + v_{it}, \qquad i = 1, \ldots, N, \quad t = 1, \ldots, T, \tag{4}$$

where $i$ represents the $i$th production unit, $t$ the $t$th time period, $x_{it}$ is a $1 \times K$ input vector and $\beta$ is a $K \times 1$ vector of parameters. If the $u_{it}$ terms are replaced by $u_i$, that is, the efficiency component is time-invariant, and if $\{v_{it}\}$ and $\{u_i\}$ are independently and identically distributed, the model is similar to the variance components models studied by Nerlove (1965), Wallace and Hussain (1969), and others except that $u_i$ is one-sided distributed. This model, henceforth referred to as model I, is the limiting case of (4) in which all inefficiency stays with the firm over time.

For model II, the other limiting case of eq. (4), it is assumed that for $t \neq t'$, $E(u_{it}u_{it'})=0$ for all $i$ and $E(u_{it}u_{jt'})=0$ for all $i \neq j$. In this case, none of the firms inefficiency stays with it over time. Estimation of this model is the same as set forth in Aigner et al. (1977) for a single cross-section and the benefits of pooled data are minimal.

Model III is the intermediate case where it is assumed that for $t \neq t'$, $E(u_{it}u_{it'})=\sigma_{tt'}$ for all $i$ and $E(u_{it}u_{jt'})=0$ for all $i \neq j$. The assumption $E(u_{it}u_{it'}) =\sigma_{tt'}$, that is, the variance and covariances depend on time periods, permits some inefficiency to stay with the firm and some which does not.

If inefficiency stays with the firm over time, it is possible that it would be learned by firms. In this case, firms choice of inputs may be correlated with the efficiency component $u_i$, thus violating the assumptions of the regression model. Indeed, it has been claimed by some investigators that capital-intensity and technical efficiency are positively related. White (1978), in his survey of the question of appropriate factor proportions in LDC manufacturing, states that the major argument in favor of capital-intensive techniques is the claim that labor-intensive alternatives 'would always use more labor *and more capital* per unit of output than would the process with the high capital–labor ratio'. This is equivalent to stating that labor-intensive firms are less technically efficient than otherwise identical firms employing more capital-intensive techniques. Analysis of covariance provides an alternative procedure for which the problem of a correlation between $u_i$ and the inputs is eliminated by the inclusion of firm dummy variables which represent a non-random but still time invariant efficiency term. In the case of model III, learning is made difficult because $u_{it}$ varies with time, nevertheless, one cannot be assured of no correlation between the $u$'s and the inputs. There is no single approach which is both unrestrictive as to the specification of the efficiency component and necessarily provides unbiased estimates of the models parameters.[1] Below, three different approaches are used to investigate the sources of inefficiency in the Indonesian weaving sector.

[1]If valid instrumental variables such as prices exist, consistent estimates of the model can be derived. Unfortunately, our data set does not include the necessary variables.

### 2.3. Estimation and testing

#### 2.3.1. Estimation of models I and II

First, let us consider the estimation of model I, where $u_i$ is time invariant. Since $u$ is one-sided distributed it has nonzero mean, which cannot be identified from the intercept in (4) without knowledge of its specific distribution. Following Aigner et al., we consider the case of $u_i$ as truncated normal[2] and $v_i \sim N(0, \sigma_v^2)$. It is well-known that in a variance components model under the assumption that both $u$ and $v$ are normal with zero mean, the generalized least squares method is asymptotically efficient in the estimation of $\beta$ [see, e.g., Maddala (1971)]. This is not the case in our specification since generalized least squares does not utilize the information of $u$'s truncation. To find efficient estimates, maximum likelihood procedures are necessary. The likelihood function is derived in the appendix.

With the specification (1), a measure of each unit's efficiency can be defined as

$$z_{it}/f(x_i, \beta)\,e^{v_{it}} \tag{5}$$

for the $i$th unit in the $t$th time period. As $v_{it}$ is unobservable, (5) is not estimable. However, mean efficiency, defined as the expected value of the ratio in (5), is estimable and is a useful index. The mean efficiency measure is simply $E(e^u)$, the moment generating function $\phi(\lambda)$ evaluated at $\lambda = 1$.

With the truncated normal distribution, the mean efficiency measure [Lee and Tyler (1978)] is

$$E(e^u) = 2\,e^{\sigma_u^2/2}(1 - \Phi(\sigma_u)), \tag{6}$$

where $\Phi$ is the standard normal cumulative density function.

For model II, $u_{it}$ is independently and identically distributed over time. The maximum likelihood approach described in Aigner et al. (1977) is directly applicable without modification. For details see Aigner et al. (1977).

#### 2.3.2. Estimation of model III

The use of maximum likelihood methods for estimating model III with time variant efficiency is precluded because of the difficulty in specifying a

[2] The density function of the truncated normal variable $u$ is

$$h(u) = (2/\sqrt{2\pi}\,\sigma_u)\exp\{-u^2/2\sigma_u^2\}, \qquad u \leqq 0.$$

Alternative one-sided distributions for $u$ rather than truncated normal distributions can also be used. Among those, Afriat (1972) and Richmond (1974) utilize the one-parameter Gamma distribution. Meeusen and van den Broeck (1977) and Aigner et al. (1977) also utilize the exponential distribution. All of these distributions have similar theoretical properties, however, the truncated normal distribution is preferred from the computational point of view.

flexible multivariate distribution for $(u_{i1},\ldots,u_{iT})$ with each component $u_{it} \leq 0$. The multivariate truncated normal distribution is a possible candidate but the implied likelihood function is computationally intractible (see appendix 2 for such a likelihood function). Instead, a set of $T$ equations are estimated by Zellner's (1962) seemingly unrelated regression subject to the constraint that slopes are equal across time periods.[3] That is, we have the system of equations

$$y_i = X_i\beta + \varepsilon_i, \qquad i=1,\ldots,N, \tag{7}$$

where

$$y_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix}, \quad X_i = \begin{bmatrix} X_{i1} \\ X_{i2} \\ \vdots \\ X_{iT} \end{bmatrix}, \quad \varepsilon_i = \begin{bmatrix} u_{i1}+v_{i1} \\ u_{i2}+v_{i2} \\ \vdots \\ u_{iT}+v_{iT} \end{bmatrix},$$

and the covariance matrix $\Omega$ of the disturbances $\varepsilon_i$ is

$$\Omega = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1T} \\ \sigma_{21} & \sigma_{22} & & \sigma_{2T} \\ \vdots & \vdots & & \vdots \\ \sigma_{T1} & \sigma_{T2} & & \sigma_{TT} \end{bmatrix} + \sigma_v^2 I_T. \tag{8}$$

The above system is then estimated by generalized least squares. While this model is the most general formulation of the variance components model with truncated disturbances, a measure of average efficiency is not readily obtained from its estimation. However, this model, in which the other two models are nested, is useful in ascertaining the robustness of the estimated coefficients of the model with a time-invariant efficiency component.

### 2.3.3. *A model test procedure*

To test the specification of models I and II as compared with the more flexible specification model III, we can use the following $\chi^2$ testing procedure provided in Jöreskog and Goldberger (1972).[4]

Let $\Omega=[\sigma_{ts}]$ be the covariance matrix of $\varepsilon_i$. $\Omega$ can be estimated as follows. Estimate each cross section equation

$$y_{it} = x_{it}\beta + \varepsilon_{it}, \qquad i=1,\ldots,N,$$

[3]An $F$-test validates this restriction. See section 5.

[4]An alternative procedure is the maximum likelihood ratio test. However, for model III, the likelihood function is intractible and hence the likelihood ratio test is precluded (see appendix 2).

by ordinary least squares and compute the estimated residuals $\hat{\varepsilon}_{ts}$. $\sigma_{ts}$ is then estimated as

$$\hat{\sigma}_{ts}=\frac{1}{N}\sum_{i=1}^{N}\hat{\varepsilon}_{it}\hat{\varepsilon}_{is}, \tag{9}$$

and $S=[\hat{\sigma}_{ts}]$ is the estimate of $\Omega$.

The covariance matrix of $\varepsilon_i$ in model I is

$$\Sigma=\sigma_u^2 ll'+\sigma_v^2 I_T, \tag{10}$$

where $l'=(1,\ldots,1)$ is a $T$ dimensional vector of ones. To test model I, we first estimate $\sigma_u^2$ and $\sigma_v^2$ by minimizing the quantity

$$\begin{aligned} G(\sigma_u^2,\sigma_v^2)&=\tfrac{1}{2}\operatorname{tr}[I-S^{-1}\Sigma]^2 \\ &=\tfrac{1}{2}\operatorname{tr}[S^{-1}(S-\Sigma)]^2. \end{aligned} \tag{11}$$

As shown in Jöreskog and Goldberger (1972), the statistics $NG(\hat{\sigma}_u^2,\hat{\sigma}_v^2)$, where $\hat{\sigma}_u^2$ and $\hat{\sigma}_v^2$ are estimates of $\sigma_u^2$ and $\sigma_v^2$ derived from the minimization of $G(\sigma_u^2,\sigma_v^2)$, is asymptotically chi-square distributed with $(T(T+1)/2)-s$ degrees of freedom where $s$ is the number of unknown parameters in $\Sigma$, i.e., $s=2$ for model I. Thus we can use these statistics to test the variance components specification in model I.

Similarly, we can test model II. For model II, $\Sigma$ is a diagonal matrix and the derived chi-square distribution has $(T(T+1)/2)-3$ degrees of freedom.

## 3. Sources of inefficiency

Maximum likelihood estimation of model I will provide an estimate of the mean level of inefficiency in the Indonesian weaving industry. However, this measure evaluates the industry as a whole, and provides no information on the inefficiency of individual firms in the sample. From a policy point of view, it is of interest to distinguish the inefficient firms from the efficient firms, and to determine whether inefficient firms share some common set of characteristics.

In development literature, efficiency in production has been linked with a number of firm attributes. The nature of the relationship between firm ownership and efficiency is probably in greater dispute than that of any other firm attribute. Foreign firms are alleged to be more efficient than private domestic firms because of greater experience in management and superior organizational structure. On the other hand, foreign firms may be inefficient because they operate in unfamiliar circumstances. Managers may be *satisficing* and maximizing variables other than profit. Wells (1973) has suggested that such behavior is relevant in the Indonesian context. In his study of a sample of Indonesian firms, he suggests that foreign firms do not simply maximize profits but are more concerned with the smoothness of operations and the engineering aesthetic, the desire of engineers for mechanical efficiency.

Morley and Smith (1977) hypothesize that foreign firms in LDC's will operate at below maximum efficiency if they adapt technology to LDC factor price ratios and market size. They assert that as foreign firms move further away from the capital–labor ratio and scale of plant used in home operations (the *domain of competence*), management efficiency falls. Thus foreign firms adopt labor-intensive techniques only at the cost of technical inefficiency, the level of which is related to the firms factor proportions and size.

Table 1

Licensed investment in the Indonesian textile sector.[a,b]

| | Foreign investment | Domestic investment |
|---|---|---|
| Projects | 97 | 459 |
| Value of investment (billions of rupiah) | 591.78 | 565.63 |
| Investment per project (billions of rupiah) | 6.10 | 1.23 |
| Investment per employee (millions of rupiah) | 6.24 | 3.12 |
| Licensed mechanical looms[c] | 17,754 | 42,517 |

[a] *Source*: Investment Coordinating Board.

[b] Based on investment applications processed by the Investment Coordinating Board from 1969 through 1977. Not all projects have been implemented.

[c] Weaving establishments only.

Foreign investment has played an important role in the development of the Indonesian textile sector over the past decade. Table 1 indicates that foreign investment contributed over one-half of all new investment licensed from 1969 to 1977. Foreign investment projects were nearly five times as large as domestic investment projects and had twice the investment per employee. Foreign as well as domestic investment were provided incentive packages that included a profit tax holiday, loss carry over, customs duty exemption for imported capital equipment and accelerated depreciation. Weaving output (in millions of meters) grew at over 9 percent per year during the 1970's. Although the share of weaving output derived from foreign firms is unknown, table 1 demonstrates that foreign investment accounted for nearly 30 percent of newly licensed mechanical looms over the 1969–1977 period.

The efficiency of production may also be related to the age of the firm.

Older firms have had more time to learn and become more experienced in their operations and thus become more efficient. In the case of foreign firms, the efficiency loss suffered through operations outside of their domain of competence may decline with time as they become more familiar with new techniques. Countering these learning effects, the durability and high replacement cost of capital result in the use of equipment by older firms which does not embody more recent technological advances. Younger firms are able to adopt the most efficient technologies available at the time of their conception.

Another firm attribute thought related to efficiency is firm size. Large firms are often considered more efficient than small firms. This has been attributed to economies with respect to organization and technical knowledge and to firm growth resulting from past efficiency.

Where pooled cross-section and time series data has been available, the traditional approach to investigating the relationship between firm attributes and efficiency has been based on analysis of covariance, which includes separate intercept terms for each firm in the estimation of a production function. Another approach, which has a more sound theoretical justification, is based on the variance components model [see Amemiya (1976)] with firm characteristics added as extra regressors. Below, variables reflecting firm ownership, size and age are investigated as sources of inefficiency by regressing the separate firm intercepts obtained from the analysis of covariance on them, and by including them as extra regressors in models I and III.

## 4. Data

Cross-section and time series data on Indonesian weaving establishments are used for the estimation of a stochastic frontier Cobb–Douglas production function.[5] Data on fifty Indonesian weaving firms for the years 1972, 1973 and 1975 were obtained from manufacturing surveys conducted by the Central Bureau of Statistics (Biro Pusat Statistik) of Indonesia. All firms in the sample used power equipment. Output was measured by value-added, capital services by electricity consumption and labor inputs by the value of total wage payments and man-months of labor provided. Other measures of capital services available include horse-power of installed machinery and the value of energy consumed. Previous research with similar Indonesian data [Pitt (1981)] found electricity consumption to be the preferred measure of capital inputs. Two different measures of labor input were used because of questions concerning the perfection of labor markets which cloud the

[5]Alternative functional specifications are conceivable. The Cobb–Douglas specification is computationally easier and has been found applicable to Indonesian data in other studies in progress.

interpretation of the results that follow. Value-added and wage payments were adjusted to constant units by deflation with appropriate price and wage indices. Information on other firm characteristics was available and used in analyzing the sources of inefficiency.

The question of whether foreign companies pay higher wages than local counterparts for equivalent labor is crucial to interpreting results on the relative efficiency of foreign owned firms. If foreign firms pay higher wages than domestic firms for equivalent units of labor, then the wage bill labor variable will consistently overestimate the labor input into foreign firms production. A dummy variable representing foreign ownership will pick up the negative impact of this overestimation and result in an underestimate of efficiency. On the other hand, if foreign firms employees tend to be more skilled than those of domestic firms, the man-months of labor variable will tend to underestimate foreign labor input and the foreign firm dummy variable will pick up the positive impact of this underestimation. Without information on skills, our two measures of labor input constitute upper and lower bounds on the correct index of labor input and thus on the coefficients of the ownership dummy variable.

Lim (1977) has studied the question of foreign/local wage differences in West Malaysia and concluded that foreign firms do pay higher wages than local firms but that the tendency to pay wages commensurate with those of their home country (i.e., independent of skills levels) is of secondary importance in explaining this difference. The most important factor according to Lim is the greater capital intensity of foreign firms. He attributes this relationship between capital intensity and wages to the more highly skilled workers needed to operate the sophisticated equipment of capital-intensive firms. However, Lim did not have any information on the quality of workers in the firms he studied and thus his qualitative decomposition of the source of the foreign/local wage differential rests on slim evidence. Morley and Smith (1977) claim that if the quality of the labor force, size of firm and product mix are controlled for, the differences in wages paid by multinationals in Brazil and their local counterparts are slight.

Unpublished data provided strong evidence that skill differences are the most important source of the foreign/local wage difference among large Indonesian weaving and spinning establishments. The 1974 Survey Upah (Wage Survey) of the Biro Pusat Statistik found that 78.4 percent of the production workers of sampled foreign owned firms on Java were classified as skilled (terdidik) while only 45.9 percent of the workers of domestically owned firms were so classified. These data also reveal that foreign firms paid skilled workers 1 percent more and unskilled workers 20 percent more than domestically owned firms. If foreign firms had paid the same wage to each skill class as did domestic firms, the difference between them in the average wage paid to all production workers would have fallen by only 24 percent. Therefore, about three quarters of the difference in the average wage paid production workers is due to the difference in the proportion of workers classified as skilled.

Even within a skill classification, workers may not be homogeneous. Musa and Hallak (1977) found strong evidence of this for a sample of Indonesian textile firms. They found partial correlation coefficients between education (in years) and a dummy variable for foreign ownership of 0.522, 0.421 and 0.506 for managers, bookkeepers and skilled operators respectively. Although results were not provided for unskilled workers, it seems likely that this relationship would extend to them as well. Thus, if the education and skill of the labor force is controlled for, differences in the wages paid by foreign and domestic weaving firms are slight. On this basis, the wage bill measure of labor input may be preferred.

The variables used are

Capital — annual consumption of electricity in kilowatt-hours
Labor 1 — annual deflated wage payments (1972 base-year).
Labor 2 — annual man-months of labor.
72*D* — time dummy variable; $72D=1$ for 1972, 0 otherwise.
73*D* — time dummy variable; $72D=1$ for 1973, 0 otherwise.
Year — year firm began production (in two digits).
Foreign — dummy variable for firm ownership; takes the value one if firm is foreign owned and zero otherwise. Firms are considered foreign-owned if foreign participation exceeds 50 percent.[6]
Size — firm size measured as total man-months (in thousands) of labor supplied over the three years observed.
72 Labor, 72 Capital, 72 Year, etc. — interaction term of variable with time dummy variable, e.g., 72 Labor $=72D \times$ Labor.

All the output and factor input variables have large variances across establishments and time periods.

## 5. Empirical results

Columns (1) and (2) of table 2 report the results of applying the variance-components model (model I) to the pooled data. In both cases, capital and labor elasticities are significantly different from zero at the 5 percent level of significance. Eq. (2), using the man-month labor input measure, has larger labor and capital elasticities.

Estimates of $\sigma_u^2$ and $\sigma_v^2$ are derived directly from the maximum likelihood procedure. In contrast to some earlier exercises [for example, Aigner et al.

[6]The only firm in our sample that had foreign participation but was not considered 'foreign' in the analysis was only 25 percent foreign owned.

Table 2

Stochastic frontier Cobb–Douglas production functions (asymptotic standard error in parenthesis).

| | Model I | | Analysis of covariance | | Model I with firm characteristics | | Model III with firm characteristics | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Constant | −2.8056 | −4.8737 | | | −4.1981 | −5.6613 | −4.5204 | −5.9103 |
| | (0.5599) | (0.6902) | | | (0.6820) | (0.8262) | (0.6496) | (0.8982) |
| 1972*D* | 0.5249 | 0.6446 | 0.4509 | 0.5430 | 0.4382 | 0.5909 | 0.5244 | 0.5917 |
| | (0.1511) | (0.1625) | (0.1676) | (0.1657) | (0.1572) | (0.1601) | (0.2037) | (0.1901) |
| 1973*D* | 0.4950 | 0.6119 | 0.4615 | 0.5884 | 0.4713 | 0.5698 | 0.4915 | 0.5735 |
| | (0.1278) | (0.1540) | (0.1557) | (0.1559) | (0.1514) | (0.1525) | (0.1381) | (0.1464) |
| Capital | 0.2357 | 0.2979 | 0.1623 | 0.1662 | 0.2083 | 0.2750 | 0.2443 | 0.2706 |
| | (0.0524) | (0.0523) | (0.0677) | (0.0659) | (0.0481) | (0.0501) | (0.0442) | (0.0416) |
| Labor 1 | 0.7512 | | 0.8024 | | 0.6809 | | 0.6867 | |
| | (0.0728) | | (0.1607) | | (0.0915) | | (0.0884) | |
| Labor 2 | | 0.8825 | | 1.1214 | | 0.7347 | | 0.7649 |
| | | (0.1019) | | (0.2074) | | (0.1181) | | (0.1169) |
| Foreign | | | | | −0.6231 | 0.0859 | −0.7217 | −0.0280 |
| | | | | | (0.3033) | (0.3145) | (0.2915) | (0.3151) |
| Year | | | | | 0.0315 | 0.0311 | 0.0249 | 0.0266 |
| | | | | | (0.0070) | (0.0074) | (0.0066) | (0.0076) |
| Size | | | | | 0.0219 | 0.0184 | 0.0166 | 0.0138 |
| | | | | | (0.0082) | (0.0102) | (0.0087) | (0.0108) |
| $\sigma_u^2$ | 0.3030 | 0.4926 | | | 0.1306 | 0.2549 | | |
| | (0.1729) | (0.2803) | | | (0.1437) | (0.1909) | | |
| $\sigma_v^2$ | 0.6022 | 0.6159 | | | 0.5594 | 0.5801 | | |
| | (0.0796) | (0.0905) | | | (0.0779) | (0.0815) | | |
| $E(\varepsilon^u)$ | 0.6772 | 0.6176 | | | 0.7663 | 0.6971 | | |
| Log likelihood value | −81.9888 | −88.1142 | | | −71.7008 | −78.3540 | | |
| $R^2$ | | | 0.7915 | 0.7986 | | | | |

(1977)] the $\sigma_u^2$ are not swamped by the $\sigma_v^2$. The use of the man-months of labor variable increases $\sigma_u^2$ slightly. The mean efficiency of the Indonesian weaving industry with this labor input measure is 61.8 percent compared to 67.7 percent with the value of labor input measure. These are comparable to the 62.5 percent average efficiency found for all Brazilian industry [Lee and Tyler (1978)], and the 55.4 and 55.8 percent for the Colombian apparel and footwear industries respectively [Tyler and Lee (1979)] but somewhat lower than than the 90.9 percent found for the French textile industry [Meeusen and van den Broeck (1977)].

Columns (3) and (4) of table 2 present the analysis of covariance estimates of the frontier production function. Although, as Maddala (1971) has shown, analysis of covariance estimates do not utilize any between group information, the approach is appropriate if it is felt that firm inefficiency is correlated with labor and capital inputs. To investigate the sources of inefficiency, separate firm intercepts obtained from the analysis of covariance estimates are regressed on the three firm characteristics: age (measured as the year the firm began production), size and ownership. The results of these regressions are

$$\begin{array}{l} \text{Firm intercept} = -2.3870 - \underset{(0.3086)}{0.7157}\,FOREIGN + \underset{(0.0074)}{0.0312}\,YEAR \\ \text{[from table 2, eq. (3)]} \\ \qquad + \underset{(0.0064)}{0.0169}\,SIZE, \qquad \bar{R}^2 = 0.2559, \end{array} \tag{12}$$

$$\begin{array}{l} \text{Firm intercept} = -7.3551 + \underset{(0.3685)}{0.2099}\,FOREIGN + \underset{(0.0093)}{0.0272}\,YEAR \\ \text{[from table 2, eq. (4)]} \\ \qquad + \underset{(0.0117)}{0.0004}\,SIZE, \qquad \bar{R}^2 = 0.1540, \end{array} \tag{13}$$

where the separate firm dummy variables are in logarithmic form. All three independent variables in (12) are significant at the 5 percent level and indicate that larger firms are more efficient than smaller firms, younger firms are more efficient than older firms and that domestically owned firms are more efficient than foreign owned firms. Only the age of firm variable is significant at the 5 percent level in eq. (13) whose dependent variable is derived from the analysis of covariance using the physical units of labor input variable. It is not surprising that the ownership variable is significant in only one of these equations as we have already argued that the two labor input variables constitute upper and lower bounds on actual input use.

Investigation of the sources of inefficiency can be performed within the variance components model by adding firm characteristics thought to be correlated with inefficiency as extra regressors in the estimated production functions. Results of such an estimation are reported in the fifth column of table 2, where firm size, age and ownership characteristics are added as extra regressors. The coefficients of these firms characteristics are of the same sign and magnitude as those found in eq. (12). A joint test comparing the MLE specification of column (1) with that of colum (5) finds that the addition of the three variables is highly significant with a $-2$ log likelihood ratio of 20.576 and the $\chi^2$ distribution with three degrees of freedom.[7] In addition, the elasticities on both labor and capital become smaller and thus returns to scale fall. This is due to the inclusion of a measure of firm size which is correlated with efficiency in the regression.

In comparing the MLE specification of columns (1) and (5) note how the estimate of $\sigma_v^2$ falls only slightly when the extra regressors are added. On the other hand, the estimate of $\sigma_u^2$ falls nearly 57 percent. These three firm characteristics thus explain more than half the variance of the permanent component and 27.6 percent of inefficiency.

Although coefficient estimates of model I and model III are very similar, it is of interest to test which of the three specifications of the efficiency term is the most appropriate. One method would be to estimate the models and calculate likelihood ratio tests. However, it was not possible to derive a likelihood function for model III and it was estimated by generalized least squares. Nevertheless, the chi-squared test devised in section 2.3.3 allows us to test the appropriateness of alternative specifications. The first null hypothesis tested is that the efficiency component is time invariant under the assumption that the random component is homoskedastic with respect to time periods. This hypothesis was rejected at the 5 percent level of significance [$\chi^2(4)=16.27$].[8] The null hypothesis that the efficiency component is time invariant under the assumption that the random component is heteroskedastic with respect to time periods was also rejected [$\chi^2(2)=6.73$].[9] Finally, the null hypothesis that variances are time independent was also rejected [$\chi^2(3)=10.43$]. Thus, by rejecting both limiting cases of the time independence of the efficiency component, model III must be the appropriate specification for the firms of our sample.

Estimates of model III with a time-variant efficiency component are presented in columns (7) and (8) of table 2. Notice that the GLS estimates of

[7] In this and all hypothesis tests which follow, results are unaffected by the choice of the labor input variable.

[8] The degree of freedom is $d=(T(T+1)/2)-2$ for model I. Since $T=3$ for our data, $d=4$.

[9] In this case, the covariance is

$$\Sigma=\sigma_u^2 ll'+\begin{bmatrix}\sigma_{v_1}^2 & \sigma_{v_2}^2 & 0 \\ 0 & & \sigma_{v_3}^2\end{bmatrix},$$

and the degree of freedom for the chi-square statistics is 2.

the coefficients of this model do not differ greatly from the MLE estimate of the time invariant efficiency component model. This indicates that results obtained from the time invariant efficiency component model are robust.

To test the presence of non-neutral technical change, and to establish the legitimacy of pooling the time series of cross sections, a model which allows different factor elasticities, firm characteristic coefficients *and firm inefficiency* across time periods is estimated. The estimated equation includes time interaction terms (72 Labor 1, 72 Capital, 72 Year, etc.) in the time-invariant efficiency component model. Based on the chi-squared test, the ten interaction terms are found to be jointly not significantly different from zero at the 5 percent level of significance [$\chi^2(10) = 16.37$]. Thus, for our data, there does not appear to be non-neutral shifts in the production function over time, and the pooling of the time series of cross sections is legitimate.

In order to demonstrate the quantitative importance of the relationship between firm characteristics and inefficiency, the firms of our sample have been grouped into quintiles according to firm characteristics and the lower and upper quintiles compared. The results of this comparison are presented in table 3. There it is seen that the youngest firms commenced production on average in the year 1971.55 while the mean first year of production for the ten oldest firms was 1942. The oldest of two firms of these vintages which were identical in every other respect would be expected to produce 47.9 percent of the output of the younger firm based on the estimate of model III with the wage bill labor input measure. If these representative firms had the mean characteristics of their quintiles, the oldest firm would produce 59.4 percent of the output of the younger firm because the younger firms are smaller and have greater foreign participation. Similar efficiency differences hold for the quintiles grouped by size. In the case of ownership, a foreign firm is expected to produce only 48.6 percent of the output of an otherwise identical domestic firm. However, since foreign firms are typically newer and larger than domestic firms, their relative efficiency increases to 84.1 percent when these other characteristics are taken into account.[10]

It is interesting to note that in two of three cases, the most efficient quintile of firms has a higher average capital–labor ratio than the less efficient quintile. Ownership is the exception, as the less efficient foreign owned

[10]The large number of weaving establishments in Indonesia seem to rule out the monopolistic argument to explain foreign firm inefficiency. The *domain of competence* argument may be applicable although data on the factor proportions used in home operations are lacking. The foreign firms in our sample are very young and it is likely that their expatriate managers learn at a different rate than indigenous managers. Lecraw (1978) found that the technical efficiency of Thai manufacturing firms increased with the experience of managers in LDC's. That foreign firms are able to survive even though they produce 16 percent less than domestic firms using the same inputs may be due to the incentives available to them under the foreign investment law and their ability to obtain capital cheaply off-shore. Note that treating the foreign/local wage differential as a labor market imperfection (i.e., use of the Labor 2 variable) causes the efficiency disadvantage of foreign firms to reappear as a labor cost disadvantage of about equal size.

Table 3

Firm characteristics and efficiency differences[a] in a sample of 50 Indonesian weaving firms.

| Groups | | Mean value of group characteristics: Age | Size | Capital[f]–labor ratio | (A) | (B)[g] |
|---|---|---|---|---|---|---|
| Age | Youngest[b,c] quintile | 1971.55 | 16.382 | 474.21 | 47.9% | 59.4% |
| | Oldest quintile | 1942.00 | 17.470 | 187.68 | | |
| Size | Largest[b] quintile | 1960.00 | 33.293 | 445.14 | 60.1% | 71.2% |
| | Smallest quintile | 1958.10 | 2.606 | 175.27 | | |
| Ownership | Domestic | 1958.50 | 12.561 | 272.57 | 48.6% | 84.1% |
| | Foreign | 1971.25 | 26.468 | 501.64 | | |
| Efficiency[d] | Most efficient quintile | 1964.20 | 15.903 | 276.89 | | 19.6%[e] |
| | Least efficient quintile | 1955.30 | 8.342 | 147.165 | | |
| Average | | 1959.52 | 13.673 | 362.62 | | |

[a](A) Output of less efficient quintile relative to more efficient quintile due only to distinguishing characteristic. (B) Output of less efficient quintile relative to more efficient quintile due to all characteristics.
[b]Includes three foreign owned firms.
[c]Quintile has eleven firms.
[d]Efficiency as determined by analysis of covariance.
[e]Total efficiency difference not just difference attributable to the three firm characteristics.
[f]Measured as total electricity consumption over three years divided by total real wage bill over three years.
[g]Based on estimate of model III with wage bill measure of labor input [table 2, column (7)].

firms have a capital–labor ratio twice that of domestic firms. The non-monotonic relationship between capital–labor ratios and efficiency is further demonstrated by noting that both the ten most efficient and ten least efficient firms as determined by analysis of covariance have capital–labor ratios below the mean.

## 6. Conclusion

In this paper, variance components models for the estimation of stochastic frontier production functions from a time series of cross-sections are introduced. Estimation methods are discussed and the models are estimated for the Cobb–Douglas case using pooled data from individual firms in the Indonesian weaving industry. Maximum likelihood estimates of a model with a time invariant efficiency component demonstrate mean efficiency for the Indonesian weaving industry of between 60 and 70 percent. An alternative specification which relaxes the assumption of a time invariant efficiency component but which permits some inefficiency to persist over time is also estimated. Statistical tests support this specification as the most appropriate one for our sample. An investigation of the sources of inefficiency find three firm characteristics, age, size and ownership, important. With these firm characteristics controlled, there is little evidence of a correlation between efficiency and capital intensity.

## Appendix 1: Derivation of the likelihood function for model I

Model I is specified as

$$y_{it}=x_{it}+u_i+v_{it}, \qquad i=1,\dots,N, \quad t=1,\dots,T,$$

where $u_i$ is *i.i.d.* one-sided distributed with truncated normal density function

$$h(u)=\frac{2}{\sqrt{2\pi}\,\sigma_u}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}, \qquad u\leqq 0;$$

$v_{it}$ is *i.i.d.* normal and $u_i$ and $v_{it}$ are independent.

Let $\varepsilon_{it}=u_i+v_{it}$ and $\varepsilon_i'=(\varepsilon_{i1},\dots,\varepsilon_{iT})$. In the following paragraph, we will derive the joint density function of $\varepsilon_i$. To simplify expressions, the subscript $i$ will be dropped. Let $g(v)$ be the density function of $v_{it}$. The joint density function $f(\varepsilon)$ of $\varepsilon_i$ can be derived from the convolution formula:

$$f(\varepsilon)=\int_{-\infty}^{0}\prod_{t=1}^{T} g(\varepsilon_t-u)h(u)\,\mathrm{d}u$$

$$=\int_{-\infty}^{0}\frac{2}{(2\Pi)^{(T+1)/2}\sigma_v^T\sigma_u}\exp\left\{-\frac{1}{2\sigma_v^2}\sum_{t=1}^{T}(\varepsilon_t-u)^2-\frac{1}{2\sigma_u^2}u^2\right\}\mathrm{d}u.$$

It is straightforward to show that the following relations hold:

(i)
$$-\frac{1}{2\sigma_v^2}\sum_{t=1}^{T}(\varepsilon_t-u)^2-\frac{1}{2\sigma_u^2}u^2$$

$$=-\frac{1}{2\sigma_*^2}\left(u-\sigma_*^2\left(\sum_{t=1}^{T}\varepsilon_t\Big/\sigma_v^2\right)\right)^2$$

$$+\tfrac{1}{2}\sigma_*^2\left(\sum_{t=1}^{T}\varepsilon_t\Big/\sigma_v^2\right)^2-\frac{1}{2\sigma_v^2}\sum_{t=1}^{T}\varepsilon_t^2,$$

where $\sigma_*^2=\sigma_u^2\sigma_v^2/(\sigma_v^2+T\sigma_u^2)$.

(ii) $$\sum_{t=1}^{T} \varepsilon_t^2 - \frac{\sigma_*^2}{\sigma_v^2}\left(\sum_{t=1}^{T} \varepsilon_t\right)^2 = \varepsilon' A \varepsilon,$$

where $A = I_T - (\sigma_u^2/\sigma_v^2 + T\sigma_u^2)ll'$, $l' = (1,\ldots,1)$ is a $T \times 1$ vector of ones and $I_T$ is the $T \times T$ identity matrix.

It follows that $f(\varepsilon)$ can be simplified to

$$f(\varepsilon) = \int_{-\infty}^{0} \frac{2}{(2\Pi)^{(T+1)/2}\sigma_v^T \sigma_u} \exp\left\{-\frac{1}{2\sigma_*^2}\left(u - \sigma_*^2\left(\sum_{t=1}^{T} \varepsilon_t \Big/ \sigma_v^2\right)\right)^2\right\}$$

$$\times \exp\left\{-\frac{1}{2\sigma_v^2}\left(\sum_{t=1}^{T} \varepsilon_t^2 - \frac{\sigma_*^2}{\sigma_v^2}\left(\sum_{t=1}^{T} \varepsilon_t\right)^2\right)\right\} \mathrm{d}u$$

$$= \frac{2\sigma_*}{\sigma_u (2\Pi)^{T/2} \sigma_v^T} \exp\left\{-\frac{1}{2\sigma_v^2} \varepsilon' A \varepsilon\right\}$$

$$\times \int_{-\infty}^{0} \frac{1}{(2\Pi)^{\frac{1}{2}} \sigma_*} \exp\left\{-\frac{1}{2\sigma_*^2}\left(u - \sigma_*^2\left(\sum_{t=1}^{T} \varepsilon_t \Big/ \sigma_v^2\right)\right)^2\right\} \mathrm{d}u$$

$$= \frac{2\sigma_u}{(\sigma_v^2 + T\sigma_u^2)^{\frac{1}{2}} (2\Pi)^{T/2} \sigma_v^T} \exp\left\{-\frac{1}{2\sigma_v^2} \varepsilon' A \varepsilon\right\}$$

$$\times \left(1 - \Phi\left(\frac{\sigma_u}{\sigma_v (\sigma_v^2 + T\sigma_u^2)^{\frac{1}{2}}} \sum_{t=1}^{T} \varepsilon_t\right)\right),$$

where $\Phi(x)$ is the standard normal c.d.f. evaluated at $x$. Hence the log-likelihood function for the pooled data is

$$\ln L = N \ln 2 - \frac{NT}{2} \ln(2\Pi) - \frac{N(T-1)}{2} \ln \sigma_v^2 - \frac{N}{2} l_n(\sigma_v^2 + T\sigma_u^2)$$

$$- \frac{1}{2\sigma_v^2} \sum_{i=1}^{N} (y_i - x_i\beta)'\left(I_T - \frac{\sigma_u^2}{\sigma_v^2 + T\sigma_u^2} ll'\right)(y_i - x_i\beta)$$

$$+ \sum_{i=1}^{N} \ln\left[1 - \Phi\left(\frac{\sigma_u}{\sigma_v (\sigma_v^2 + T\sigma_u^2)^{\frac{1}{2}}} \sum_{t=1}^{T} (y_{it} - x_{it}\beta)\right)\right]$$

$$
\begin{aligned}
&= N \ln 2 - \frac{NT}{2} \ln (2\Pi) - \frac{N(T-1)}{2} \ln \sigma_v^2 - \frac{N}{2} \ln (\sigma_v^2 + T\sigma_u^2) \\
&\quad - \frac{T}{2\sigma_v^2} \sum_{i=1}^{N} \left( \frac{y_i' y_i}{T} - 2 \frac{y_i' x_i}{T} \beta + \beta' \frac{x_i' x_i}{T} \beta \right) \\
&\quad + \frac{T^2 \sigma_u^2}{2\sigma_v^2 (\sigma_v^2 + T\sigma_u^2)} \sum_{i=1}^{N} (\bar{y}_i - \bar{x}_i \beta)^2 \\
&\quad + \sum_{i=1}^{N} \ln \left[ 1 - \Phi \left( \frac{T\sigma_u}{\sigma_v (\sigma_v^2 + T\sigma_u^2)^{\frac{1}{2}}} (\bar{y}_i - \bar{x}_i \beta) \right) \right],
\end{aligned}
$$

where $x_i$ is a $T \times K$ matrix, $y_i$ is a $T \times 1$ vector and $\bar{y}_i$, $\bar{x}_i$ are the sample means of $y$ and $x$ for the $i$th unit. It is interesting to observe from the above likelihood function that the sample means and second moments of $(y_i, x_i)$ for $i$ are sufficient statistics for our model.

Taking first derivatives,

$$
\begin{aligned}
\frac{\partial \ln L}{\partial \beta'} &= \frac{T}{\sigma_v^2} \sum_{i=1}^{N} \left( \frac{y_i' x_i}{T} - \beta' \frac{x_i' x_i}{T} \right) - \frac{T^2 \sigma_u^2}{\sigma_v^2 (\sigma_v^2 + T\sigma_u^2)} \sum_{i=1}^{N} (\bar{y}_i - \bar{x}_i \beta) \bar{x}_i \\
&\quad + \frac{T\sigma_u}{\sigma_v (\sigma_v^2 + T\sigma_u^2)^{\frac{1}{2}}} \sum_{i=1}^{N} \frac{\phi(\xi_i)}{1 - \Phi(\xi_i)} \bar{x}_i,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \ln L}{\partial \sigma_u^2} &= \frac{-NT}{2(\sigma_v^2 + T\sigma_u^2)} + \frac{T^2}{2(\sigma_v^2 + T\sigma_u^2)^2} \sum_{i=1}^{N} (\bar{y}_i - \bar{x}_i \beta)^2 \\
&\quad - \frac{T\sigma_v}{2\sigma_u (\sigma_v^2 + T\sigma_u^2)^{3/2}} \sum_{i=1}^{N} \frac{\phi(\xi_i)}{1 - \Phi(\xi_i)} (\bar{y}_i - \bar{x}_i \beta),
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \ln L}{\partial \sigma_v^2} &= -\frac{N(T-1)}{2\sigma_v^2} - \frac{N}{2(\sigma_v^2 + T\sigma_u^2)} + \frac{T}{2\sigma_v^4} \\
&\quad \times \sum_{i=1}^{N} \left( \frac{y_i' y_i}{T} - 2 \frac{y_i' x_i}{T} \beta + \beta' \frac{x_i' x_i}{T} \beta \right) \\
&\quad - \frac{T^2 \sigma_u^2 (2\sigma_v^2 + T\sigma_u^2)}{2\sigma_v^4 (\sigma_v^2 + T\sigma_u^2)^2} \sum_{i=1}^{N} (\bar{y}_i - \bar{x}_i \beta)^2 \\
&\quad + \frac{T\sigma_u (2\sigma_v^2 + T\sigma_u^2)}{2\sigma_v^3 (\sigma_v^2 + T\sigma_u^2)^{3/2}} \sum_{i=1}^{N} \frac{\phi(\xi_i)}{1 - \Phi(\xi_i)} (\bar{y}_i - \bar{x}_i \beta),
\end{aligned}
$$

where $\phi$ and $\Phi$ are standard normal density and distribution functions evaluated at $\xi_i$ with $\xi_i = (T\sigma_u/\sigma_v(\sigma_v^2 + T\sigma_u^2)^{\frac{1}{2}})(\bar{y}_i - \bar{x}_i\beta)$.

The second derivatives can also be derived but they are relatively complicated. To find the maximum likelihood estimates, various numerical algorithms which require only first derivatives, such as the Davidson–Fletcher–Powell (DFP) algorithm, can be used.

## Appendix 2: Derivation of the likelihood function for model III

For completeness of the model specification, in this appendix the expression of the likelihood function is derived for reference. Model III is specified as

$$y_{it} = x_{it}\beta + u_{it} + v_{it}, \qquad i = 1,\ldots,N, \quad t = 1,\ldots,T,$$

where $u_i' = (u_{i1},\ldots,u_{iT})$ is independently identically distributed across observations $i$ with multivariate truncated normal density function

$$h(u_i) = (2\Pi)^{-T/2}|\Sigma|^{-\frac{1}{2}}\exp\{-\tfrac{1}{2}u_i'\Sigma^{-1}u_i\}/P_0, \qquad u_{it} \leqq 0 \quad \text{for all } t.$$

where $P_0 = \int_{-\infty}^{0}\cdots\int_{-\infty}^{0}(2\Pi)^{-T/2}|\Sigma|^{-\frac{1}{2}}\exp\{-\frac{1}{2}u_i'\Sigma^{-1}u_i\}\,\mathrm{d}u_i$ is the probability of $u_i \leqq 0$, which is a function of the parameter matrix $\Sigma$; $v_{it}$ is *i.i.d.* normal $N(0,\sigma_v^2)$ and $u_i$ and $v_{it}$ are independent.

Let $\varepsilon_{it} = u_i + v_{it}$ and $\varepsilon_i' = (\varepsilon_{i1},\ldots,\varepsilon_{iT})$. The joint density function $f(\varepsilon)$ of $\varepsilon$ (subscript $i$ is dropped for simplicity) is

$$f(\varepsilon) = \int_{-\infty}^{0}\cdots\int_{-\infty}^{0}\prod_{t=1}^{T}\frac{1}{\sqrt{2\Pi}\,\sigma_v}\exp\left\{-\frac{1}{2\sigma_v^2}(\varepsilon_t - u_t)^2\right\}\cdot h(u)\,\mathrm{d}u_1,\ldots,\mathrm{d}u_T$$

$$= \frac{1}{(2\Pi)^T\sigma_v^T|\Sigma|^{\frac{1}{2}}P_0}\int_{-\infty}^{0}\cdots\int_{-\infty}^{0}\exp\left\{-\frac{1}{2\sigma_v^2}(\varepsilon'\varepsilon - 2\varepsilon'u + u'u)\right\}$$

$$\times\exp\{-\tfrac{1}{2}u'\Sigma^{-1}u\}\,\mathrm{d}u_1,\ldots,\mathrm{d}u_T$$

$$= \frac{1}{(2\Pi)^T\sigma_v^T|\Sigma|^{\frac{1}{2}}P_0}\exp\left\{-\frac{1}{2\sigma_v^2}\varepsilon'\varepsilon\right\}$$

$$\times\int_{-\infty}^{0}\cdots\int_{-\infty}^{0}\exp\left\{-\tfrac{1}{2}\left(u'\Omega^{-1}u - 2\frac{\varepsilon'}{\sigma_v^2}u\right)\right\}\mathrm{d}u_1,\ldots,\mathrm{d}u_T,$$

where $\Omega^{-1}=\Sigma^{-1}+(1/\sigma_v^2)I$. The above expression can further be simplified with the following identity:

$$\exp\left\{-\tfrac{1}{2}\left(u'\Omega^{-1}u-2\frac{\varepsilon'}{\sigma_u^2}u\right)\right\}$$

$$=\exp\left\{\frac{1}{2}\frac{\varepsilon'}{\sigma_v^2}\Omega\frac{\varepsilon}{\sigma_v^2}-\frac{1}{2}\left(u-\Omega\frac{\varepsilon}{\sigma_v^2}\right)'\Omega^{-1}\left(u-\Omega\frac{\varepsilon}{\sigma_v^2}\right)\right\}.$$

Hence

$$f(\varepsilon)=\frac{|\Omega|^{\frac{1}{2}}}{(2\Pi)^{T/2}\sigma_v^T|\Sigma|^{\frac{1}{2}}P_0}\exp\left\{\frac{1}{2\sigma_v^2}\varepsilon'\left(\frac{1}{\sigma_v^2}\Omega-I\right)\varepsilon\right\}\cdot P(\varepsilon),$$

where

$$P(\varepsilon)=\int_{-\infty}^{0}\cdots\int_{-\infty}^{0}(2\Pi)^{-T/2}|\Omega|^{-\frac{1}{2}}$$

$$\times\exp\left\{-\tfrac{1}{2}\left(u-\Omega\frac{\varepsilon}{\sigma_v^2}\right)'\Omega^{-1}\left(u-\Omega\frac{\varepsilon}{\sigma_v^2}\right)\right\}\mathrm{d}u_1,\ldots,\mathrm{d}u_T$$

is the joint probability of a multivariate normal variate $N(\Omega(\varepsilon/\sigma_v^2),\Omega)$ with each component less than or equal to zero.

Hence the log-likelihood function for the pooled data is

$$\ln L=-\frac{NT}{2}(\ln(2\Pi)+\sigma_v^2)-\frac{N}{2}\ln|\Sigma|-\frac{N}{2}\ln\left|\Sigma^{-1}+\frac{1}{\sigma_v^2}I\right|-N\ln P_0$$

$$+\frac{1}{2\sigma_v^2}\sum_{i=1}^{N}(y_i-x_i\beta)'\left(\frac{1}{\sigma_v^2}\Omega-I\right)(y_i-x_i\beta)$$

$$+\sum_{i=1}^{N}\ln P(y_i-x_i\beta).$$

This likelihood is difficult to evaluate since the quantites $P_0$ and $P(y_i-x_i\beta)$ involve $T$-dimensional numerical integrals which need to be evaluated numerically.

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